Department of Mathematical and Physical Sciences

The 2008 Colloquium

Numerical Mathematics - The Wisdom People Use Every Day

Runchang Lin, Ph.D.

Department of Mathematical and Physical Sciences
Texas A&M International University

Outline

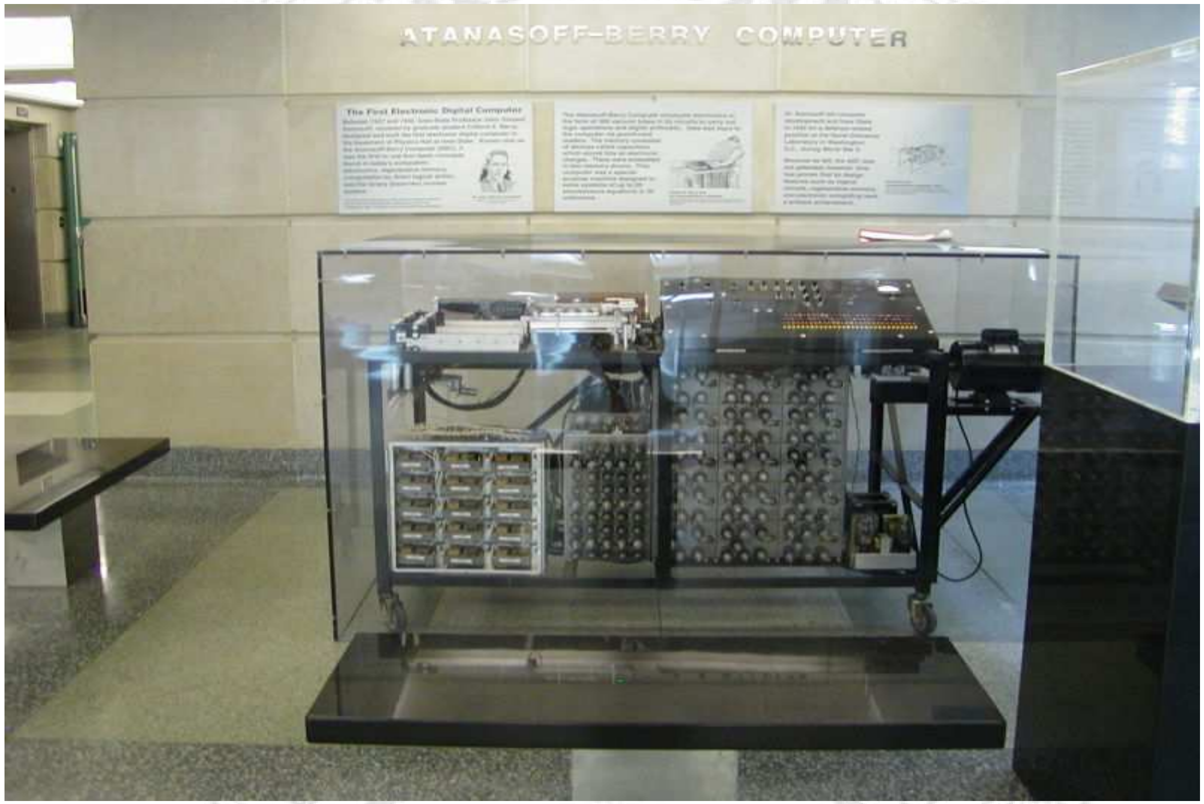
- ⋆ Is your electronic computer/calculator reliable?
- ⋆ Why numerical mathematics?
- ⋆ Scientific computing in general
- ⋆ Examples of research projects

1. Is your electronic computer/calculator reliable?

QUESTION: When was the first electronic computer invented?

1. Is your electronic computer/calculator reliable?

QUESTION: When was the first electronic computer invented?



Atanasoff-Berry Computer (1942), Iowa State University

`http://en.wikipedia.org/wiki/Atanasoff%E2%80%93Berry_Computer`

QUESTION: When was the first electronic calculator invented?

QUESTION: When was the first electronic calculator invented?

Mathatron Model 4-24/8-48 (1962), Mathatronics, Inc.

http://www.classiccmp.org/calcmuseum/calc.htm

http://www.oldcalculatormuseum.com/index.html

EXAMPLE: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$

EXAMPLE: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$

1. Compute $\sum_{n=1}^{10^{15}} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \cdots + \frac{1}{(10^{15}-1)^2} + \frac{1}{(10^{15})^2}$

```
>> sm=0; for i=1:10^15, sm=sm+1/i^2; end, sm
sm =
      1.64493405783458
```

The error is about 0.00000000901365, which cannot be reduced by this scheme.

EXAMPLE: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$

EXAMPLE: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$

2. Compute $\displaystyle\sum_{n=1}^{10^{15}} \frac{1}{n^2} = \frac{1}{(10^{15})^2} + \frac{1}{(10^{15}-1)^2} + \cdots + \frac{1}{2^2} + \frac{1}{1^2}$

```
>> sm=0;for i=1:10^15, sm=sm+1/(10e+15-i+1)^2;end,sm
sm =
      1.64493406684823
```

EXAMPLE: $$\begin{bmatrix} 1 & 10^{15}-1 \\ -1 & 10^{15}+1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10^{15} \\ 10^{15} \end{bmatrix}$$

EXAMPLE: $$\begin{bmatrix} 1 & 10^{15}-1 \\ -1 & 10^{15}+1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10^{15} \\ 10^{15} \end{bmatrix}$$

In MATLAB we put

```
>> A = [1, 10^15-1; -1, 10^15+1];
>> b = [10^15; 10^15];
>> inv(A)*b
```

EXAMPLE: $\begin{bmatrix} 1 & 10^{15}-1 \\ -1 & 10^{15}+1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10^{15} \\ 10^{15} \end{bmatrix}$

In MATLAB we put

```
>> A = [1, 10^15-1; -1, 10^15+1];
>> b = [10^15; 10^15];
>> inv(A)*b
ans =
      0.8750
      1.0000
```

2. WHY NUMERICAL MATHEMATICS?

- ⋆ When analytical solution is theoretically impossible; e.g.

$$\int_0^1 e^{x^2}\, dx = ?$$

- ⋆ When analytical solution is practically impossible; e.g.

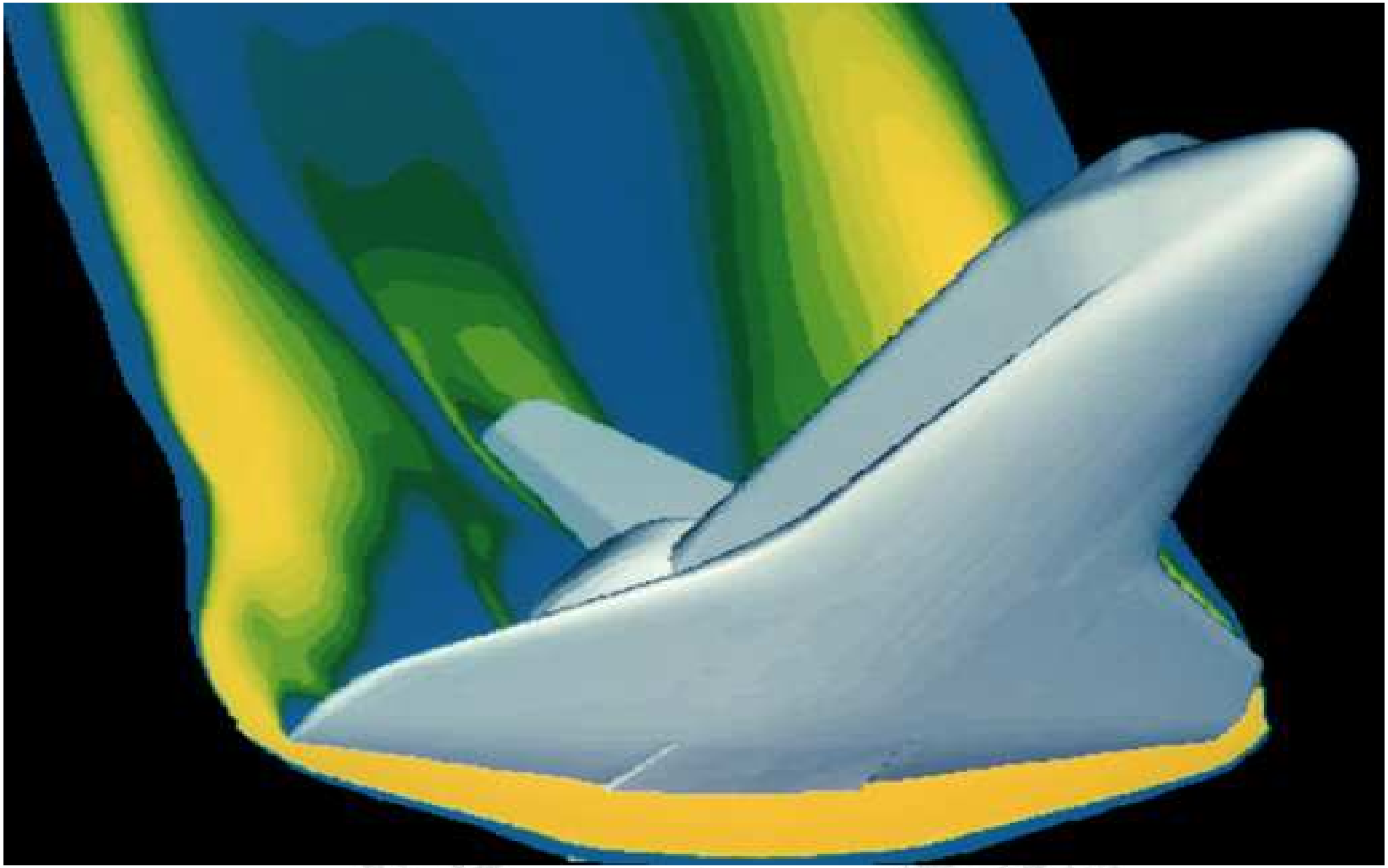
Solve $A\overrightarrow{x} = \overrightarrow{b}$ with $A_{10^{10}\times 10^{10}}$

- ⋆ When computational power is insufficient; e.g.

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad \mathcal{O}(n^2)$$

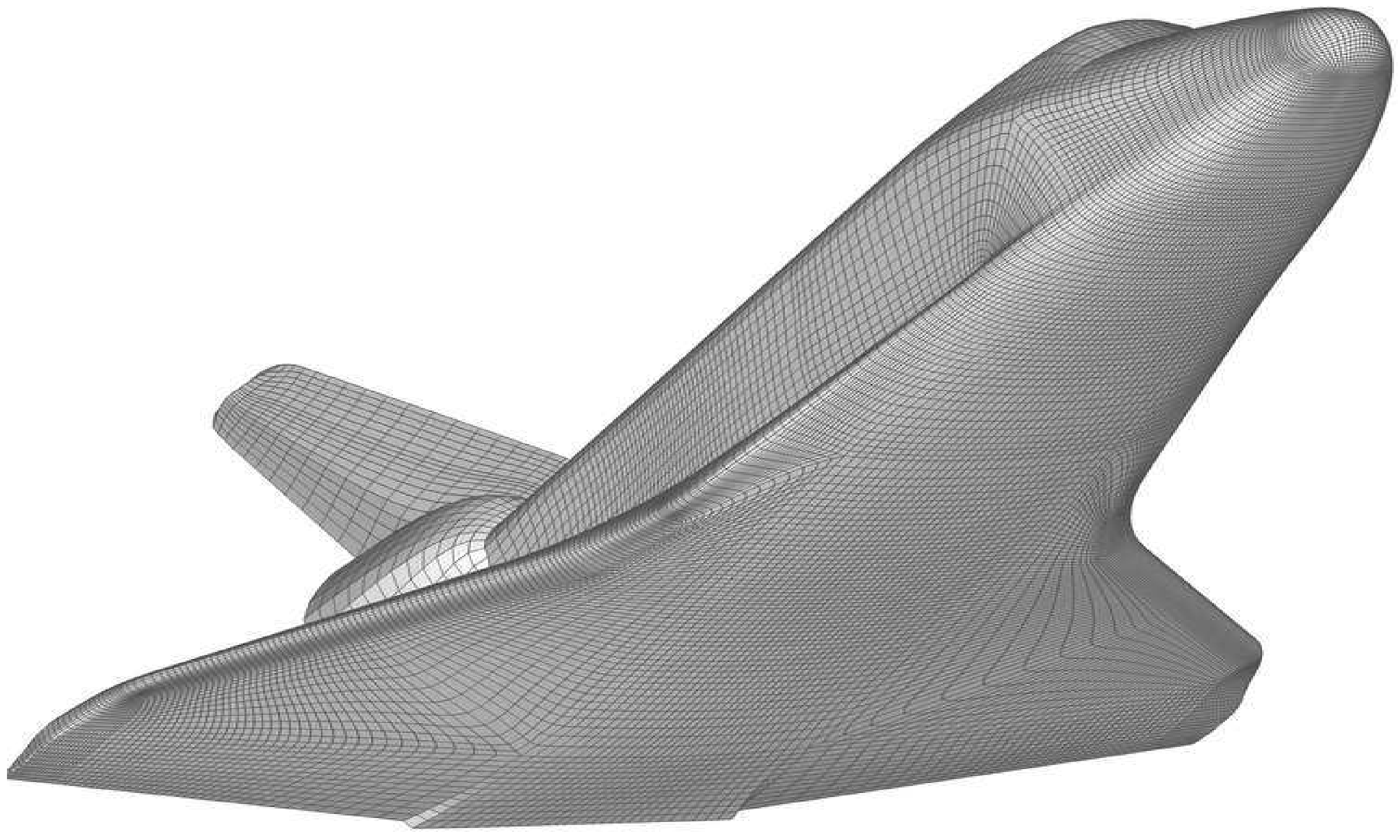
$$(\cdots((a_n x + a_{n-1})x + a_{n-2})x + \cdots + a_1)x + a_0 \quad \mathcal{O}(n)$$

Computer simulation of high velocity air ¤ow around the space shuttle during re-entr



http://en.wikipedia.org/wiki/Image:CFD_Shuttle.jpg

Computational grids



http://libmesh.sourceforge.net/wiki/index.php/Image: ...

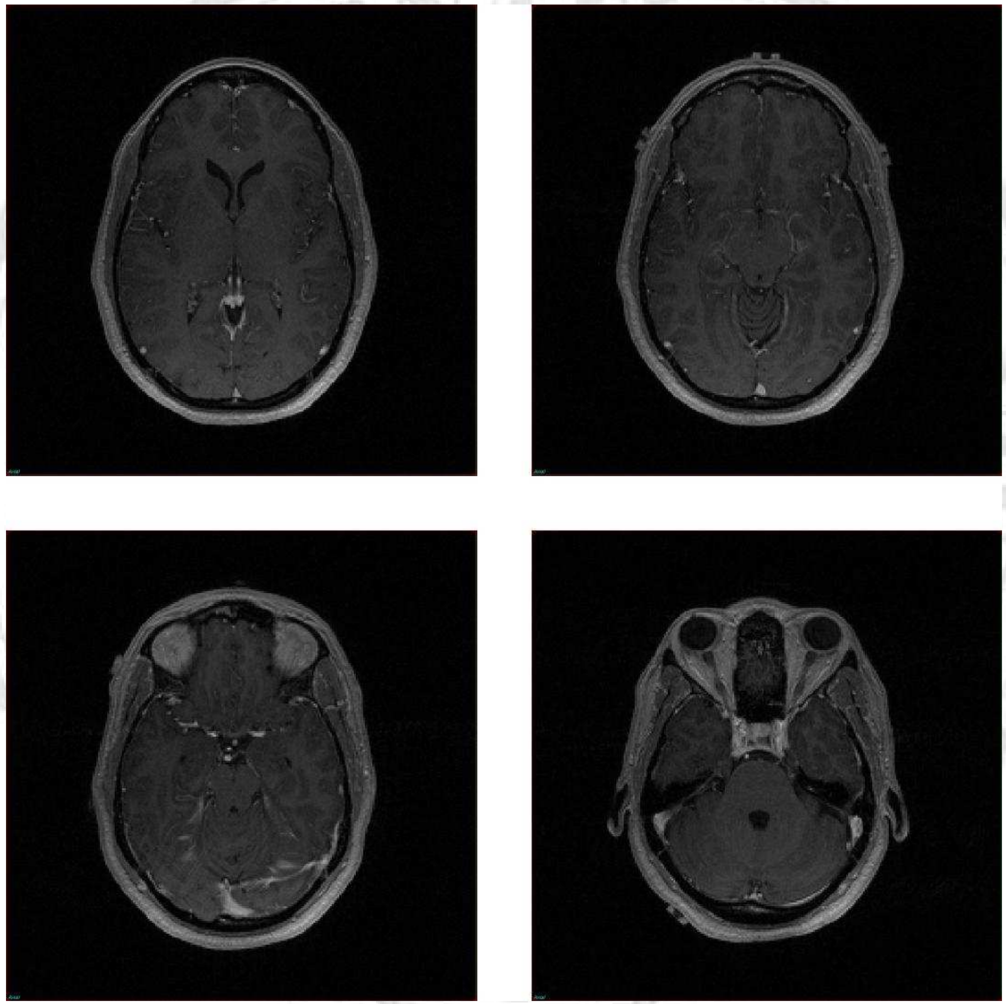
Benkirk_orbiter_schem_upper_tail.png

NUMERICAL MATHEMATICS IN DAILY LIFE

⋆ signal (image, audio, and video) processing

NUMERICAL MATHEMATICS IN DAILY LIFE

- computer graphics



NUMERICAL MATHEMATICS IN DAILY LIFE

* weather prediction

NUMERICAL MATHEMATICS IN DAILY LIFE

- ⋆ cryptography (e.g. ATM cards, computer passwords, and electronic commerce)

3. Scientific computing in general

- ⋆ *Applied mathematics* - to transform a physical problem into a mathematical problem. It is also called *modeling*.
- ⋆ *Numerical analysis* - to study algorithms and their properties.

4. EXAMPLES OF CONDUCTED/CONDUCTING PROJECTS

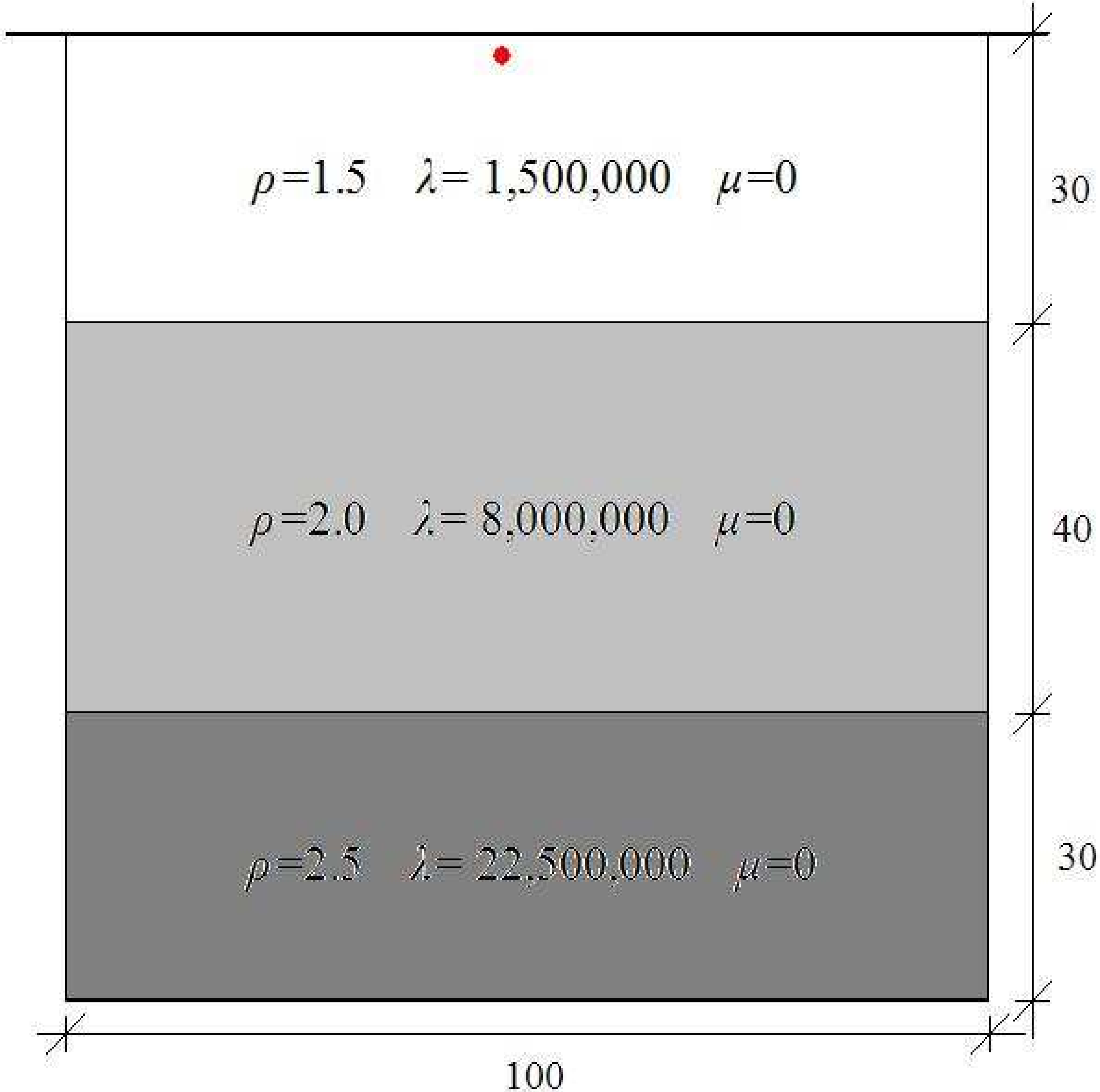
Example 1: Computer simulation of seismic wave propagation

This is a multi-physical process. Here, we consider the elastic wave equations

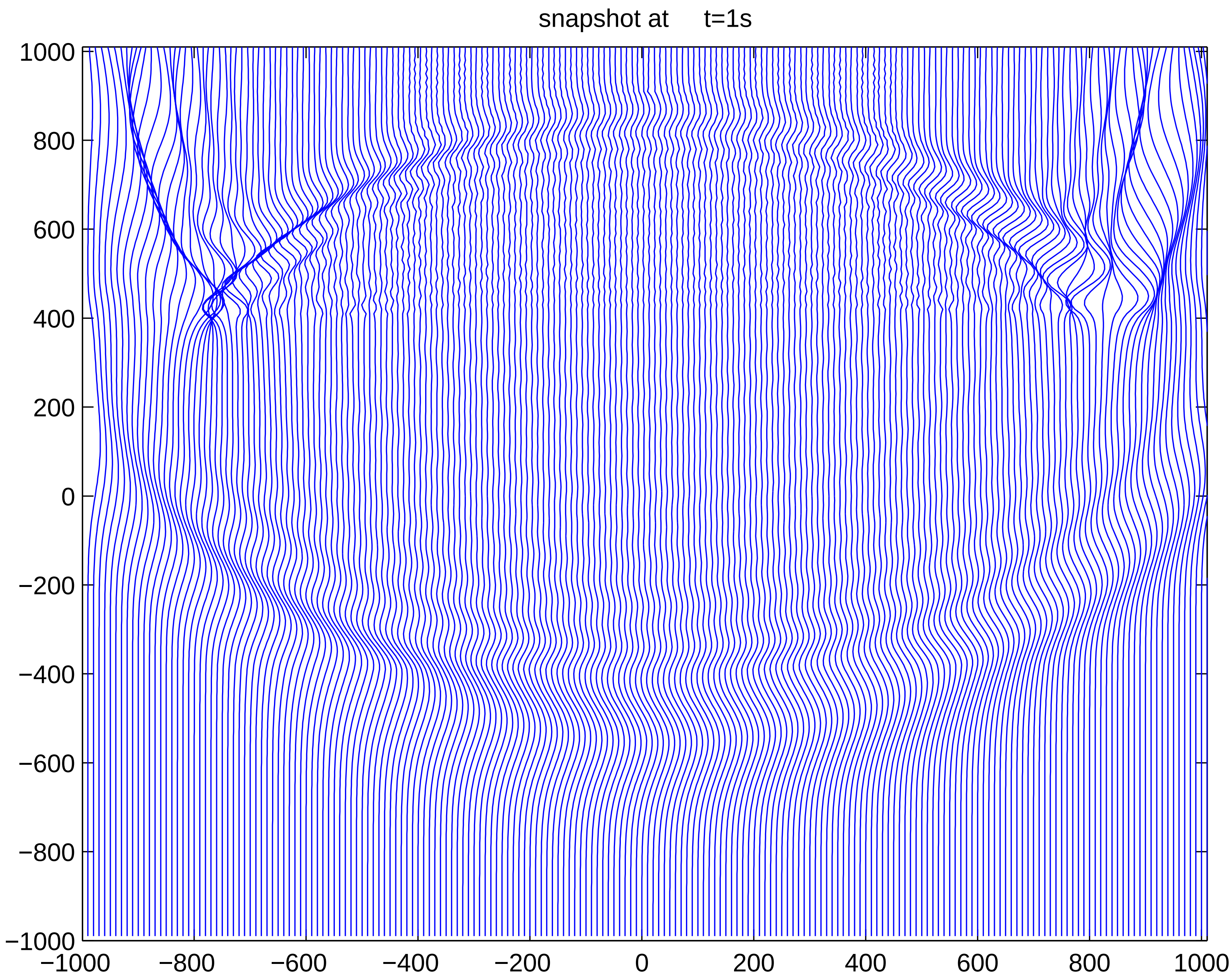
$$\rho \begin{pmatrix} u \\ v \end{pmatrix}_{tt} = \begin{pmatrix} \lambda + 2\mu & 0 \\ 0 & \mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{xx} + \begin{pmatrix} \mu & 0 \\ 0 & \lambda + 2\mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{yy} + \begin{pmatrix} 0 & \lambda + \mu \\ \lambda + \mu & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{xy}$$

with Clayton-Engquist absorbing boundary conditions. Here $\rho(x, y)$ is the density of the medium, $\lambda(x, y)$ and $\mu(x, y)$ are the Lamé parameters, u and v are the x- and y-displacements, respectively.

Computation region and parameters



snapshot at $t = 1.00$s



Example 2: Consider the 1D singularly perturbed problem

$$\begin{cases} -\epsilon u''(x) + cu(x) = f(x) \text{ on } (0,1) \\ u(0) = u(1) = 0 \end{cases} \tag{1}$$

where $0 < \epsilon \ll 1$ is the perturbation parameter, $c > 0$ is a constant, the functions f is smooth. The differential equation (1) depends on the ϵ and whose solution approaches a discontinuous limit as $\epsilon \to 0$.

Exact Solution to a Singularly Perturbed Problem

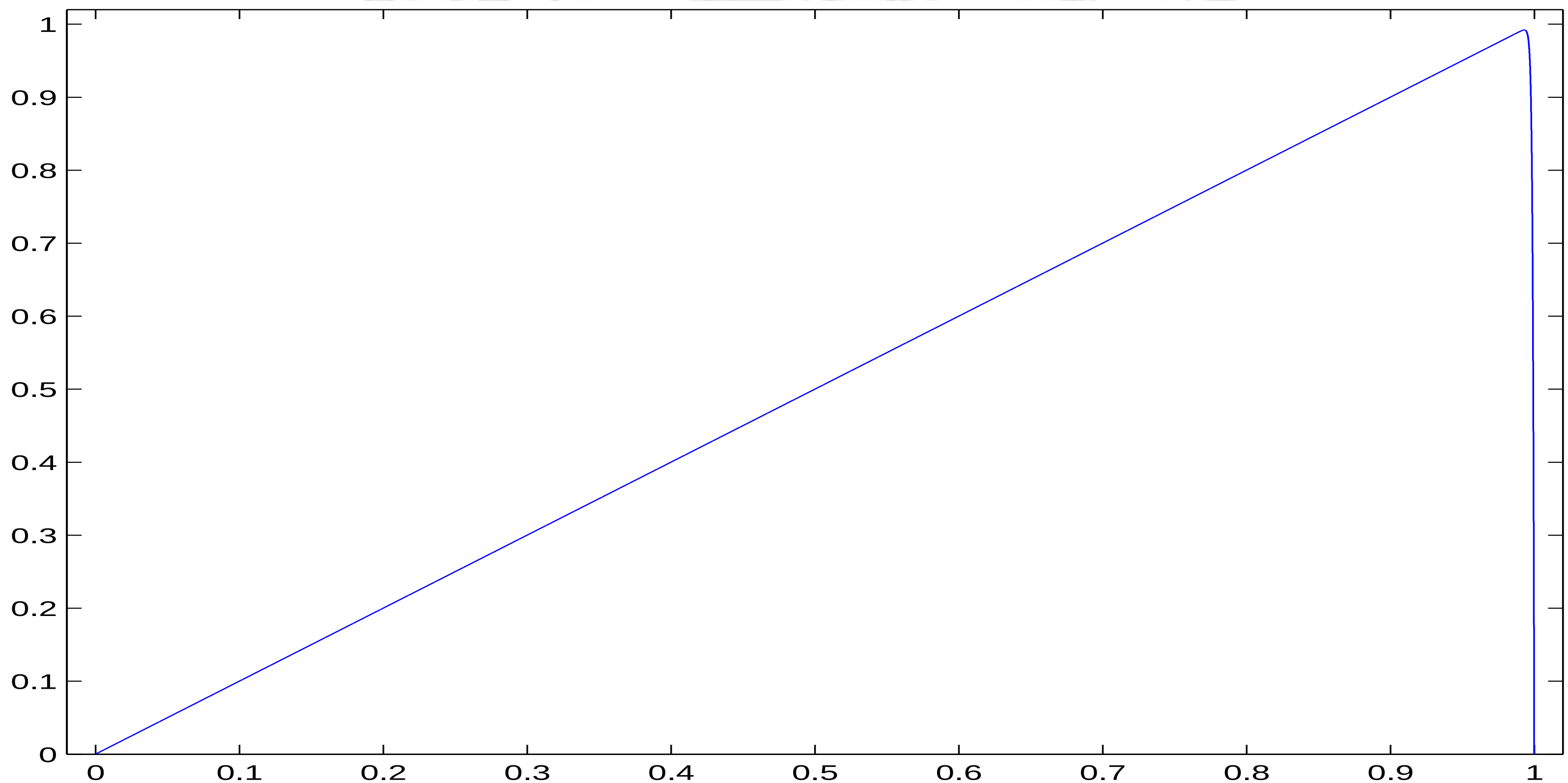
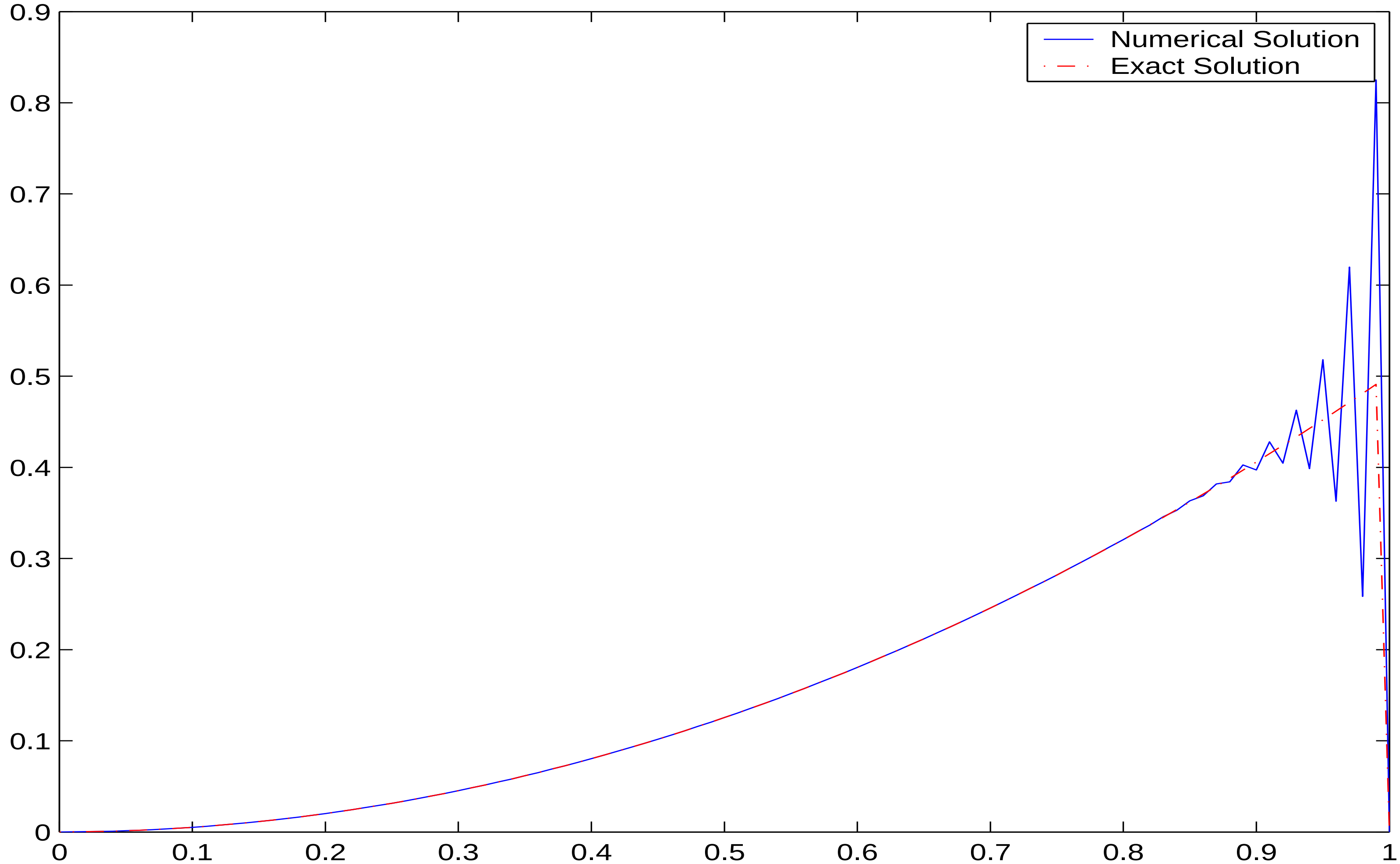
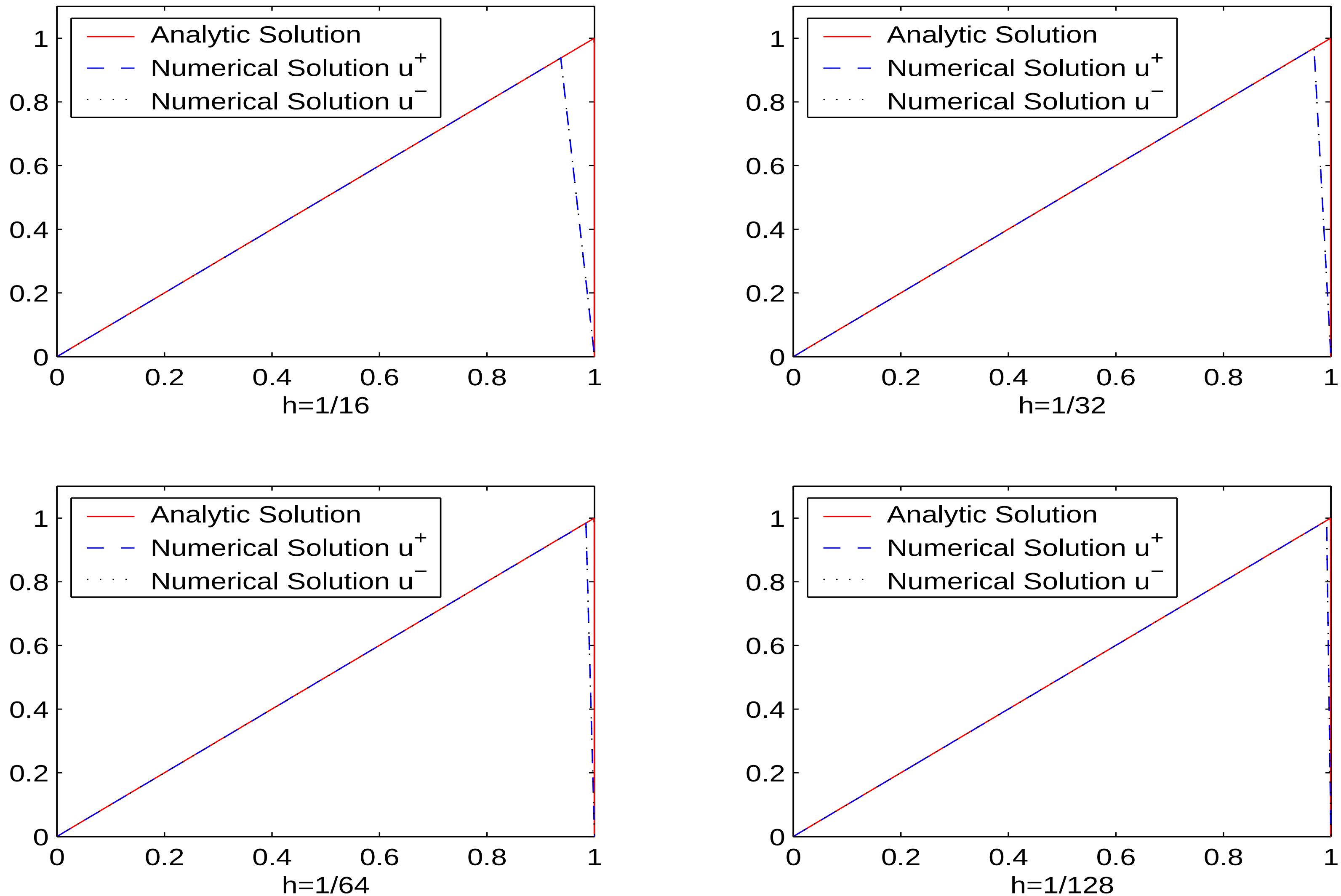


Figure 1. $\epsilon = 10^{-6}$, $c = 1$, $f(x) = x$

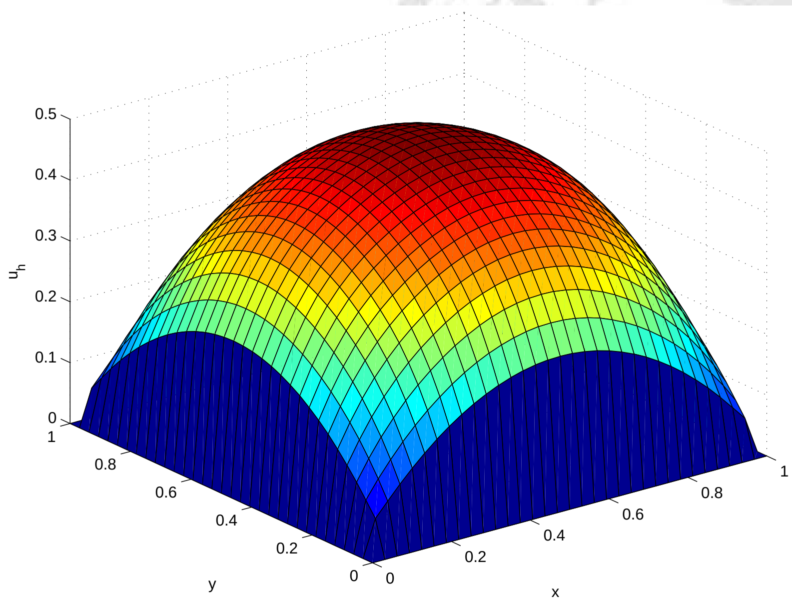
Classical Finite Element Methods Fail

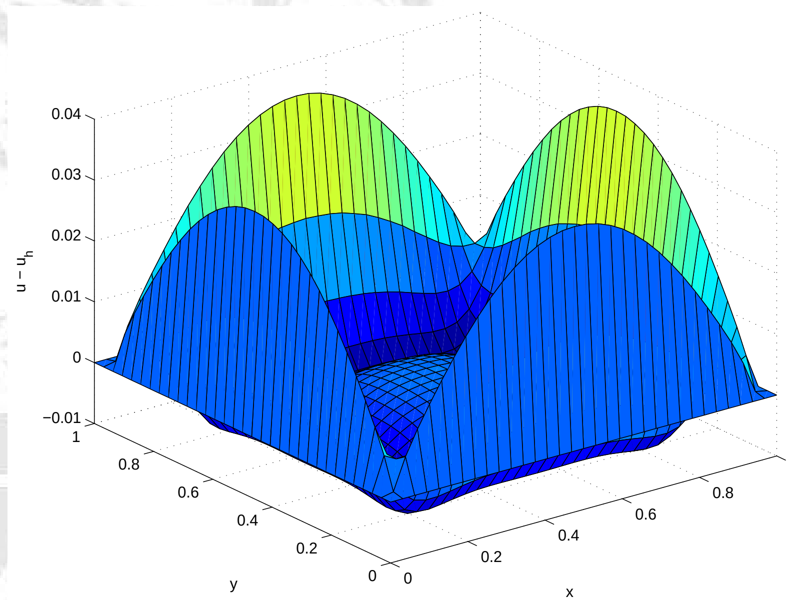


Numerical results by a discontinuous Galerkin least-squares £nite element method

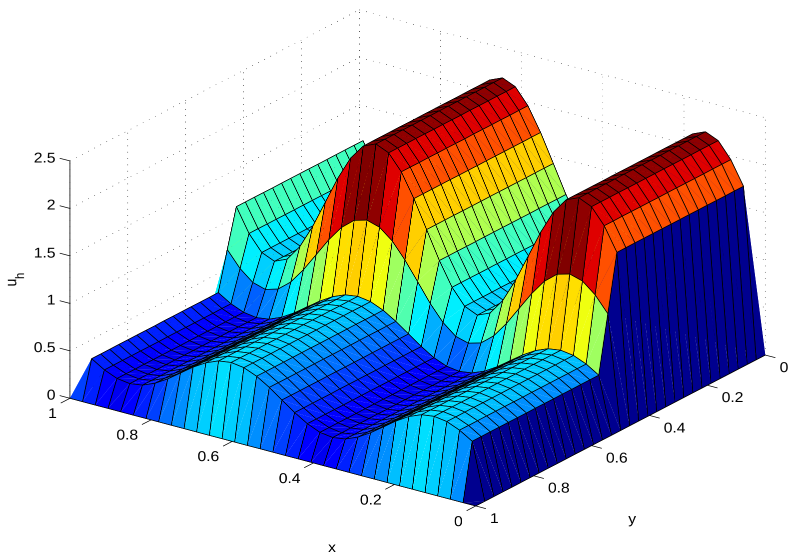


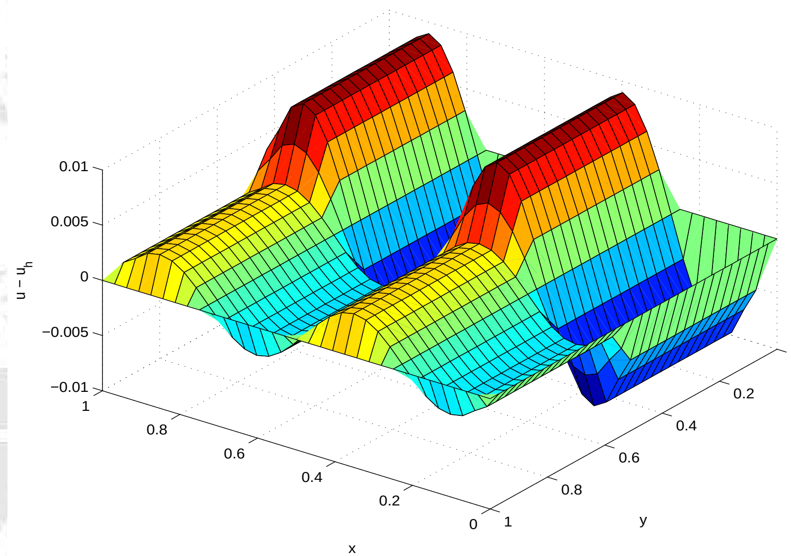
A 2D example - numerical solutions and errors





Another 2D example - numerical solutions and errors





Thank you!