

Department of Mathematical and Physical Sciences

The 2008 Colloquium

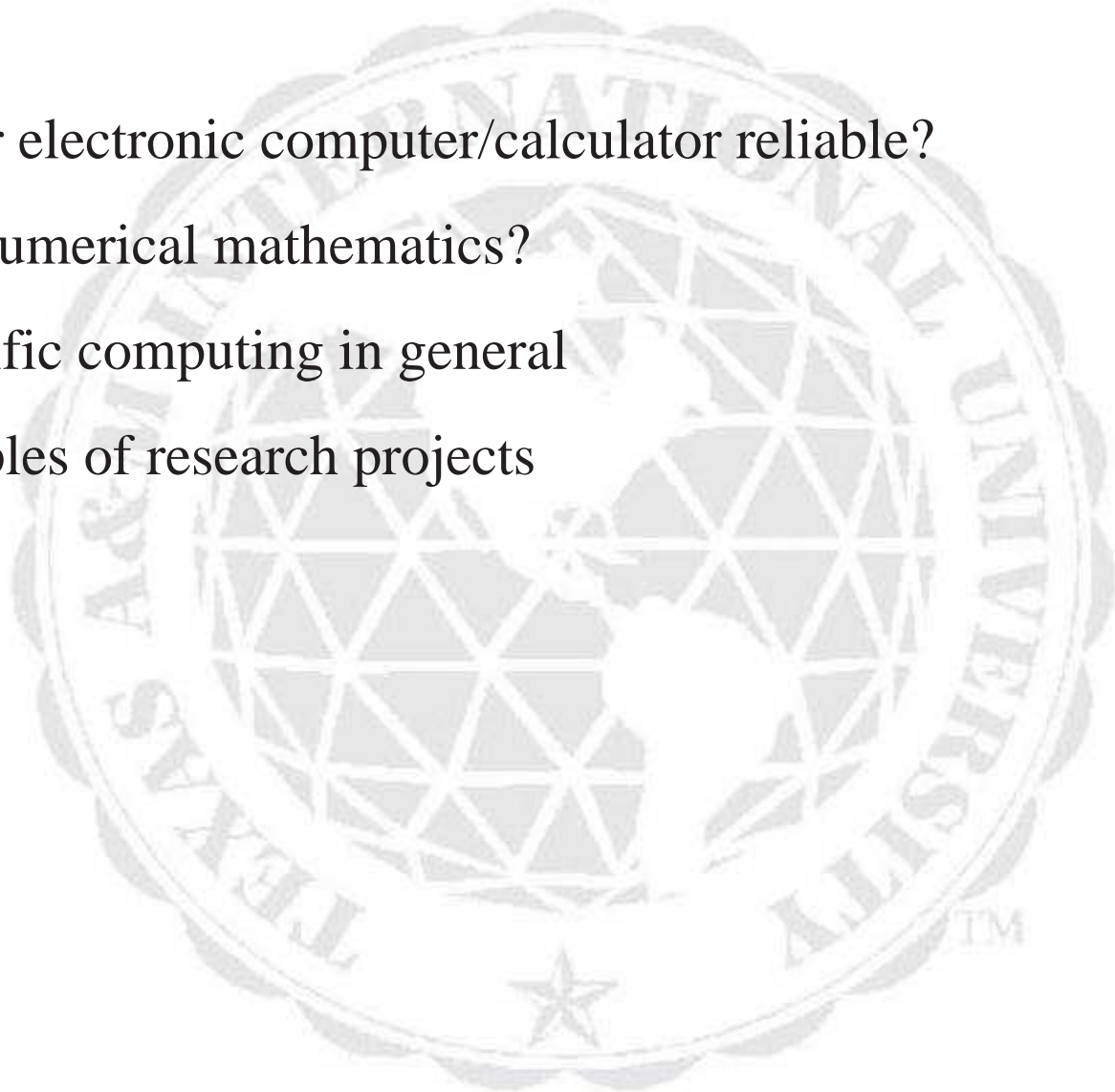
**Numerical Mathematics - The Wisdom
People Use Every Day**

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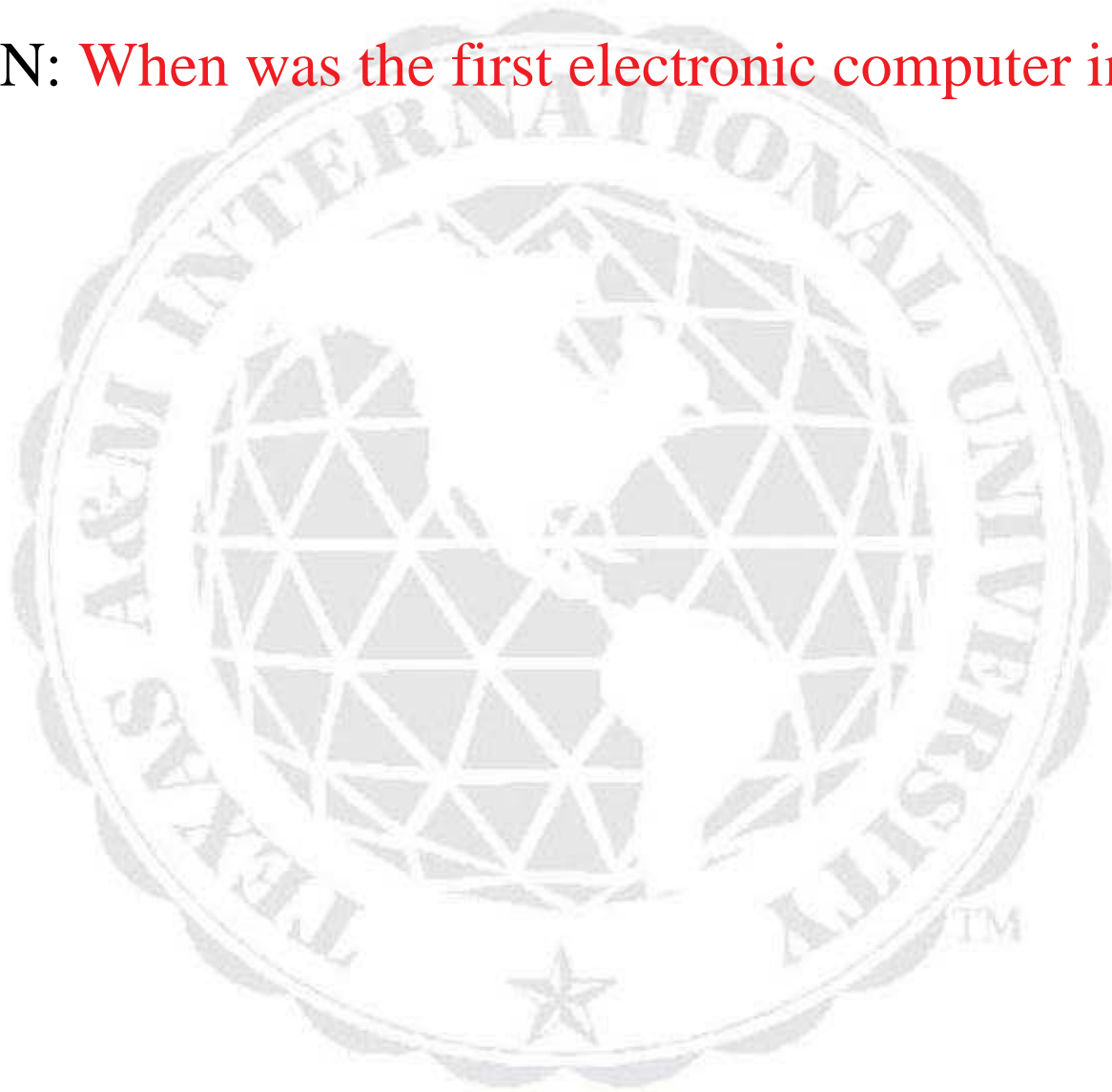
Outline

- ★ Is your electronic computer/calculator reliable?
- ★ Why numerical mathematics?
- ★ Scientific computing in general
- ★ Examples of research projects



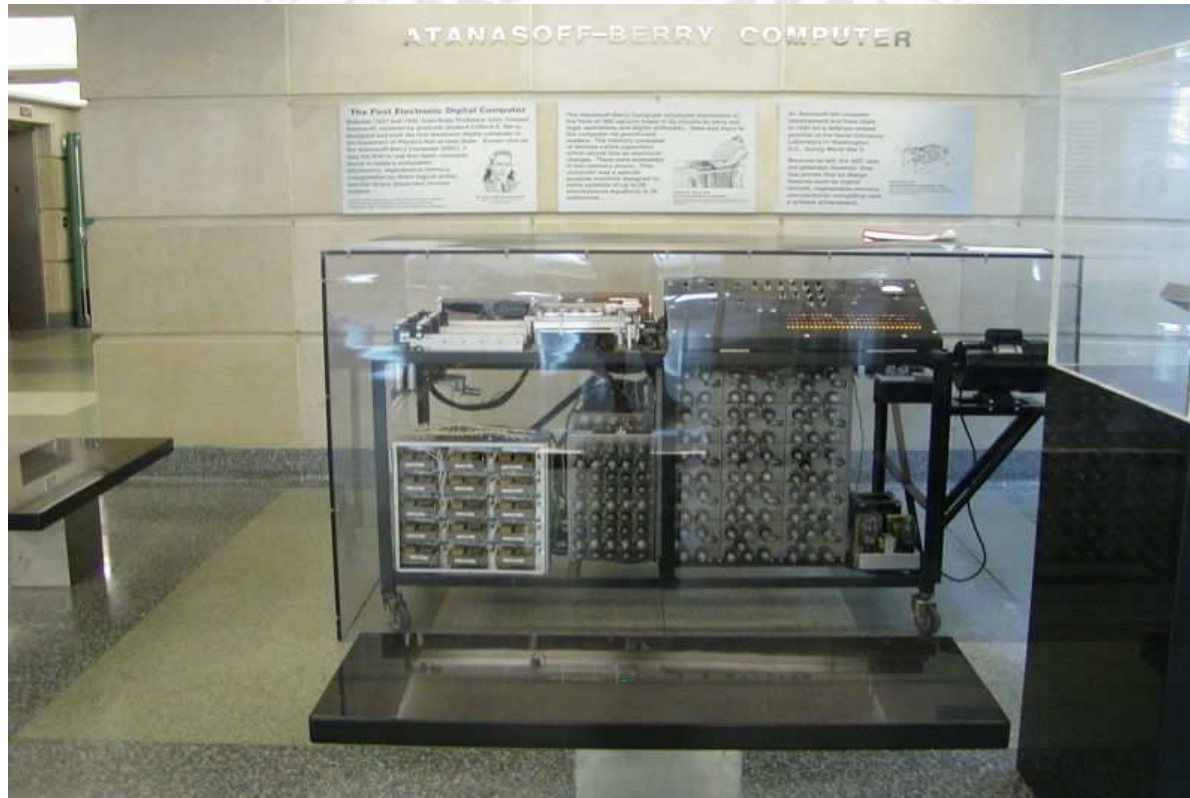
1. IS YOUR ELECTRONIC COMPUTER/CALCULATOR RELIABLE?

QUESTION: When was the first electronic computer invented?



1. IS YOUR ELECTRONIC COMPUTER/CALCULATOR RELIABLE?

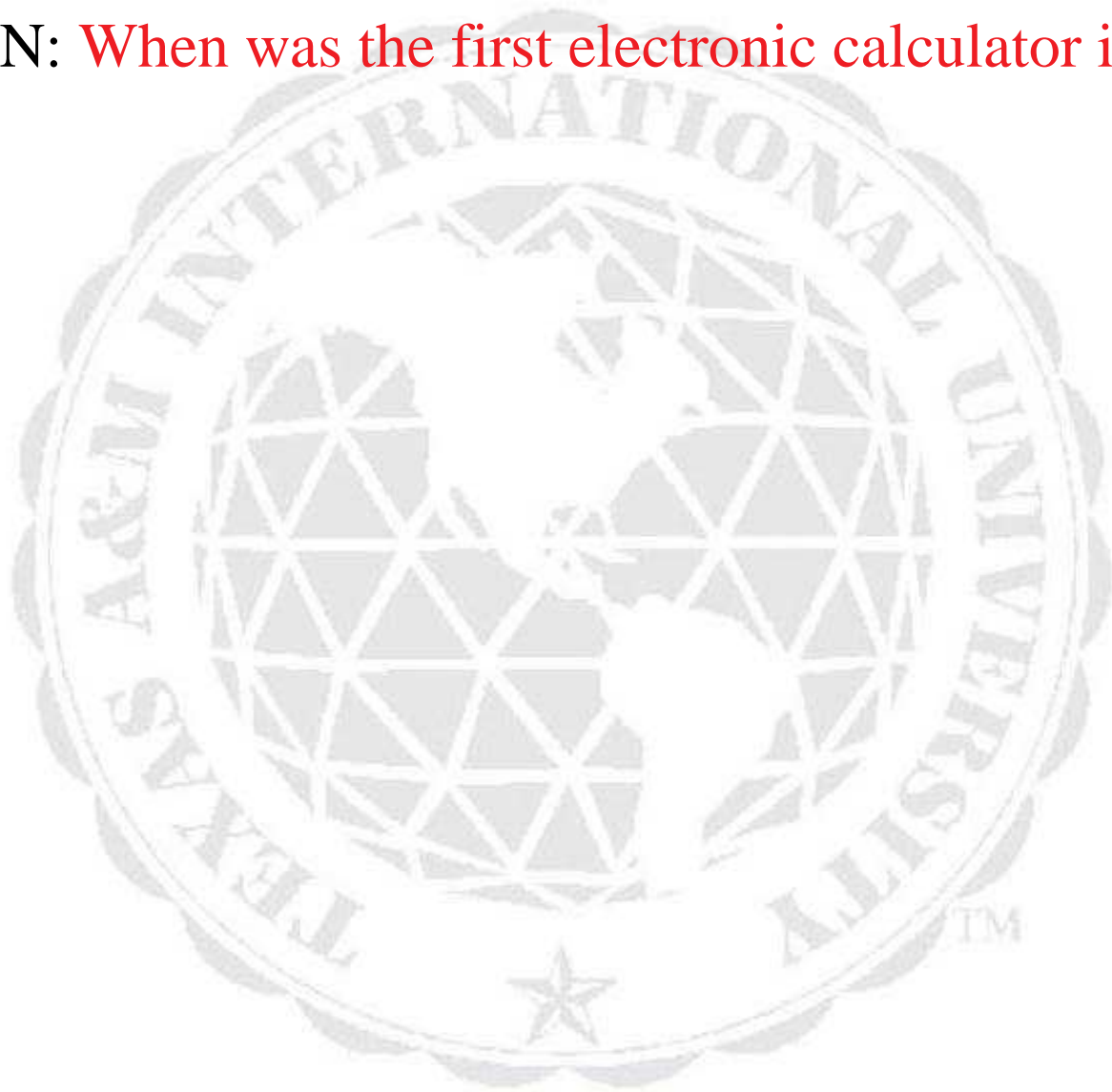
QUESTION: When was the first electronic computer invented?



Atanasoff-Berry Computer (1942), Iowa State University

http://en.wikipedia.org/wiki/Atanasoff%E2%80%93Berry_Computer

QUESTION: When was the first electronic calculator invented?



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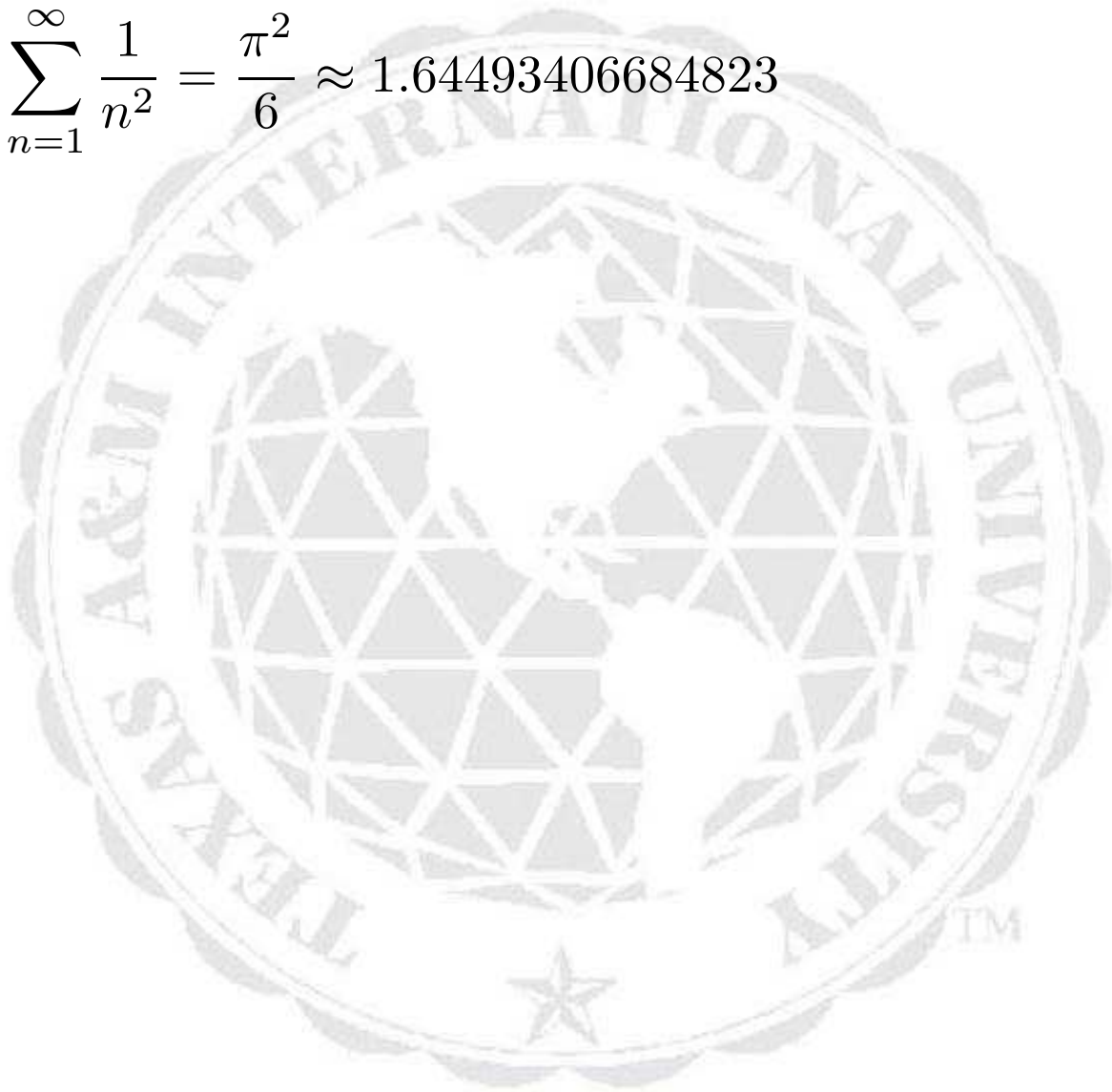


Mathatron Model 4-24/8-48 (1962), Mathatronics, Inc.

<http://www.classiccmp.org/calcmuseum/calc.htm>

<http://www.oldcalculatormuseum.com/index.html>

EXAMPLE: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$



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1. Compute $\sum_{n=1}^{10^{15}} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \dots + \frac{1}{(10^{15}-1)^2} + \frac{1}{(10^{15})^2}$

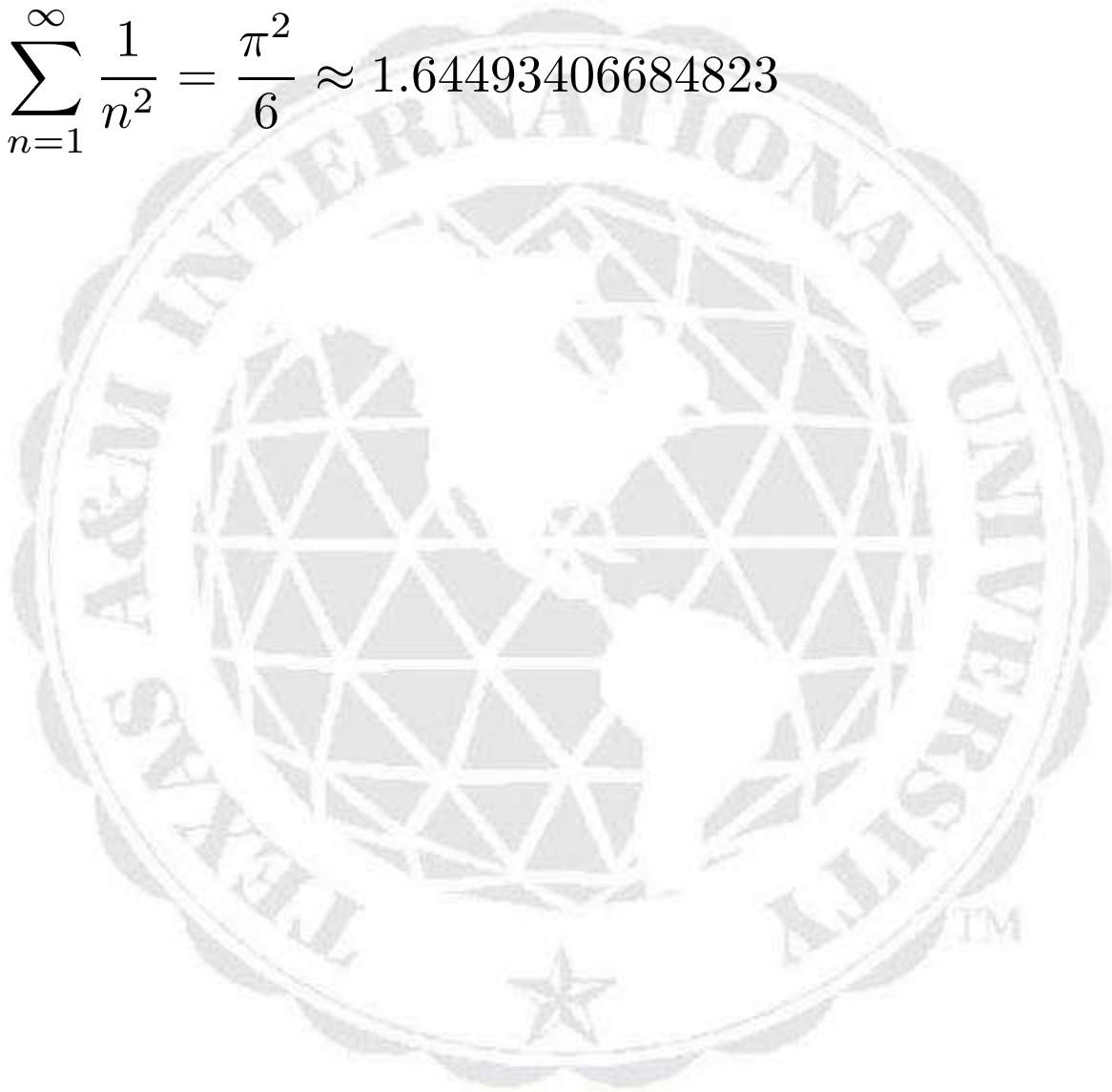
```
>> sm=0; for i=1:10^15, sm=sm+1/i^2; end, sm
```

```
sm =
```

```
1.64493405783458
```

The error is about **0.00000000901365**, which cannot be reduced by this scheme.

EXAMPLE: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$

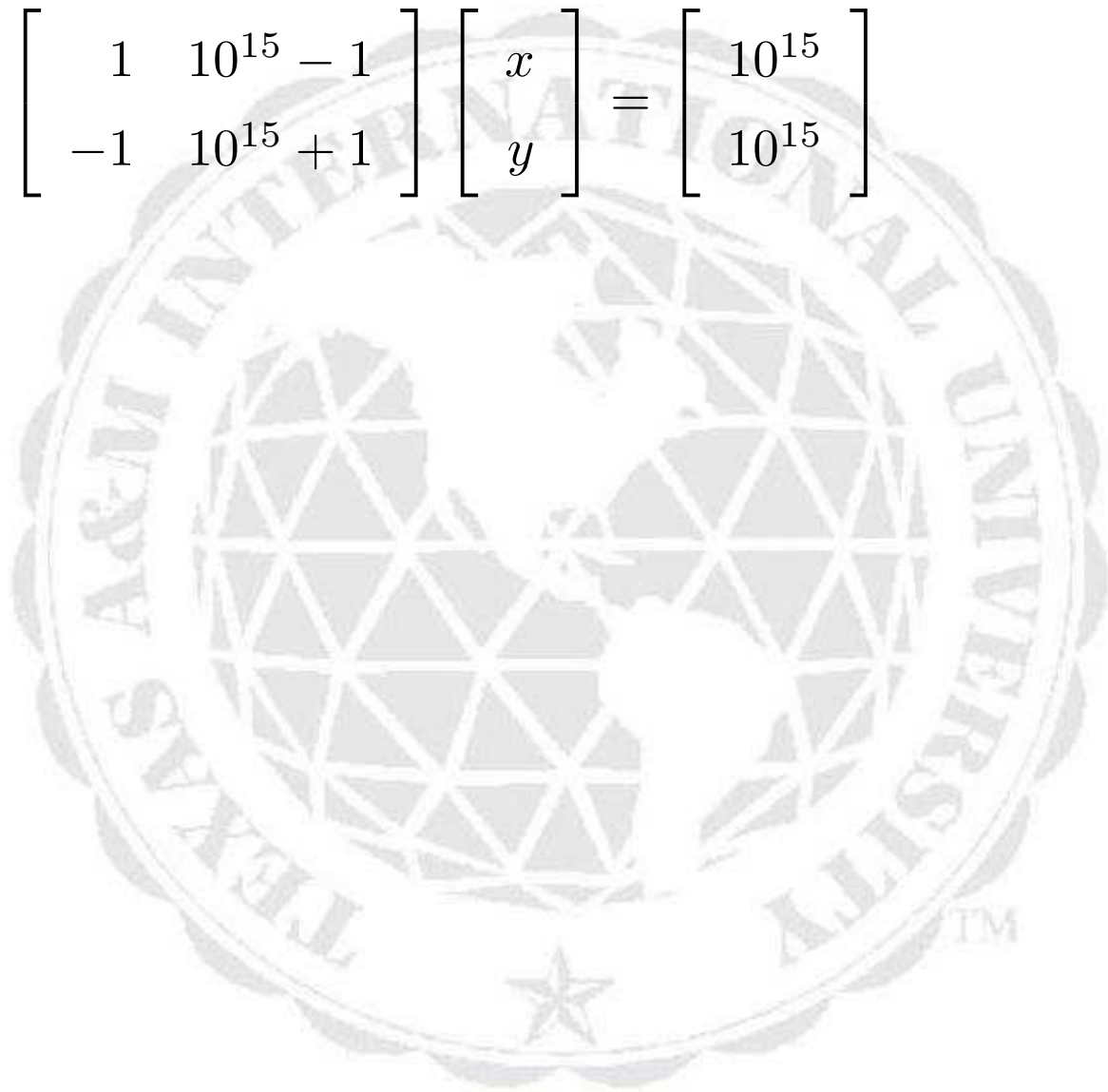


EXAMPLE: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \approx 1.64493406684823$

2. Compute $\sum_{n=1}^{10^{15}} \frac{1}{n^2} = \frac{1}{(10^{15})^2} + \frac{1}{(10^{15}-1)^2} + \dots + \frac{1}{2^2} + \frac{1}{1^2}$

```
>> sm=0;for i=1:10^15, sm=sm+1/(10e+15-i+1)^2;end,sm  
sm =  
1.64493406684823
```

EXAMPLE:
$$\begin{bmatrix} 1 & 10^{15} - 1 \\ -1 & 10^{15} + 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10^{15} \\ 10^{15} \end{bmatrix}$$



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$$\begin{bmatrix} 1 & 10^{15} - 1 \\ -1 & 10^{15} + 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10^{15} \\ 10^{15} \end{bmatrix}$$

In MATLAB we put

```
>> A = [1, 10^15-1; -1, 10^15+1];  
>> b = [10^15; 10^15];  
>> inv(A)*b
```

EXAMPLE:
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In MATLAB we put

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>> A = [1, 10^15-1; -1, 10^15+1];
```

```
>> b = [10^15; 10^15];
```

```
>> inv(A)*b
```

```
ans =
```

```
0.8750
```

```
1.0000
```

2. WHY NUMERICAL MATHEMATICS?

- ★ When analytical solution is **theoretically impossible**; e.g.

$$\int_0^1 e^{x^2} dx = ?$$

- ★ When analytical solution is **practically impossible**; e.g.

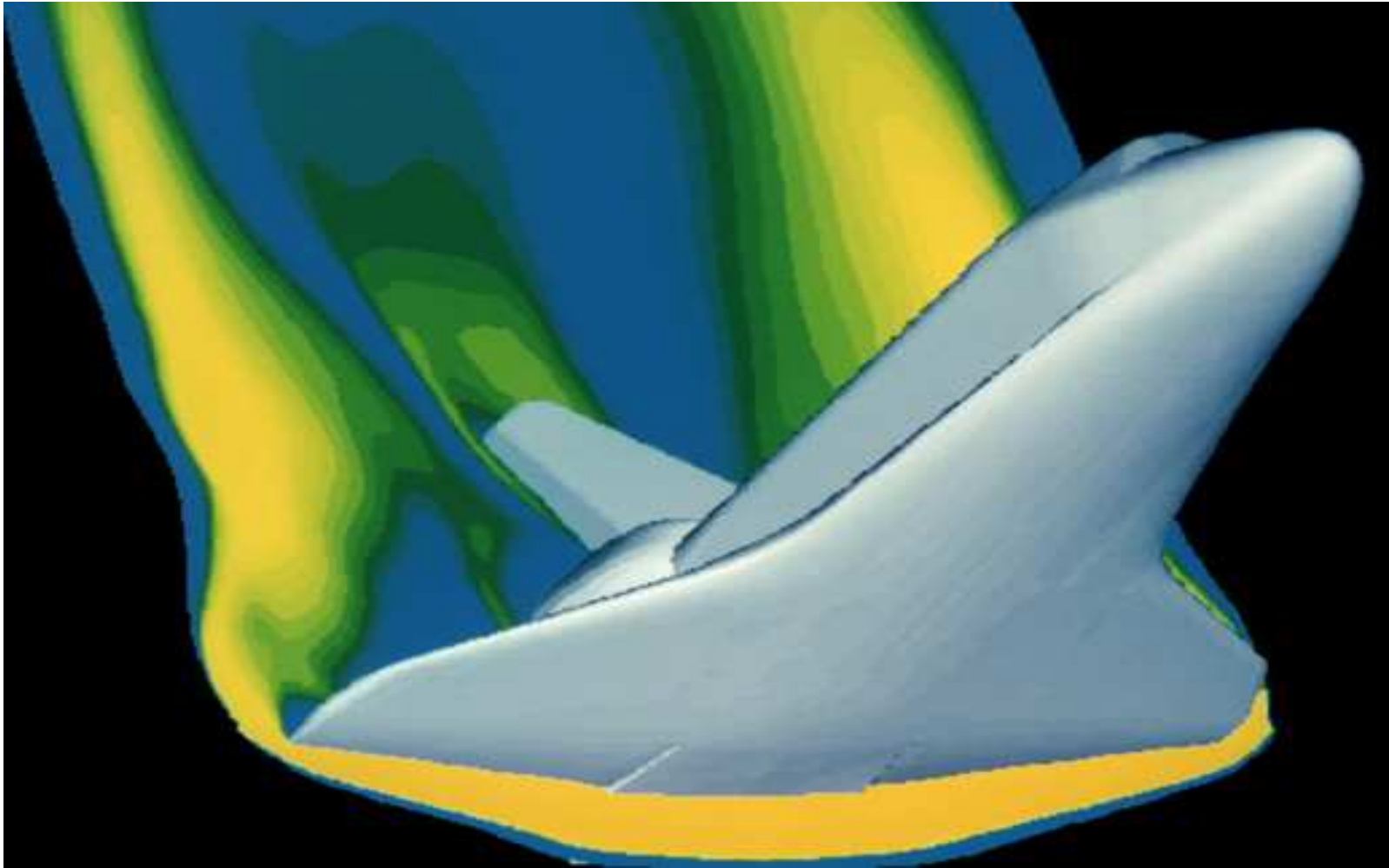
$$\text{Solve } A\vec{x} = \vec{b} \text{ with } A_{10^{10} \times 10^{10}}$$

- ★ When computational power is insufficient; e.g.

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad \mathcal{O}(n^2)$$

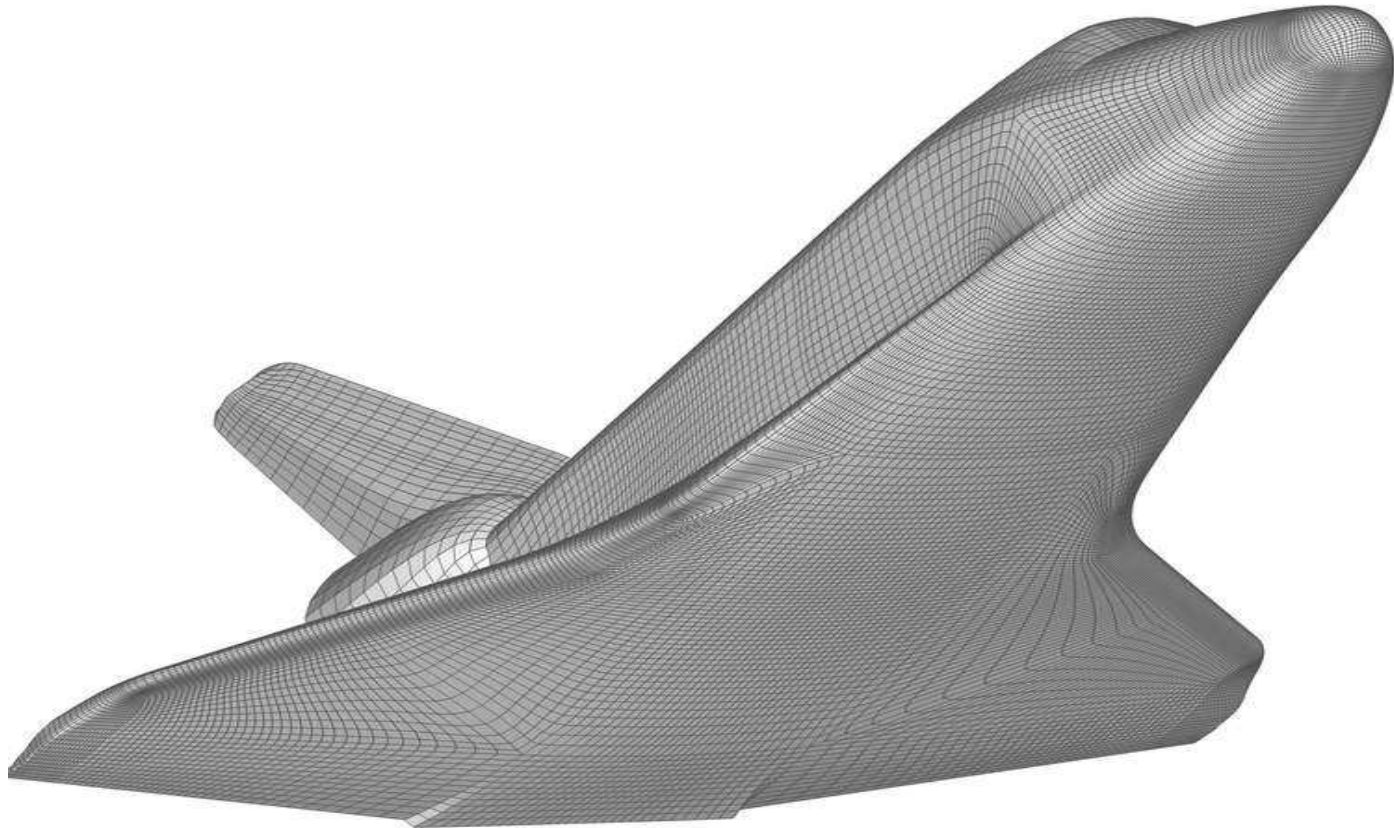
$$(\cdots ((a_n x + a_{n-1})x + a_{n-2})x + \cdots + a_1)x + a_0 \quad \mathcal{O}(n)$$

Computer simulation of high velocity air flow around the space shuttle during re-entr



http://en.wikipedia.org/wiki/Image:CFD_Shuttle.jpg

Computational grids



[http://libmesh.sourceforge.net/wiki/index.php/Image: ...](http://libmesh.sourceforge.net/wiki/index.php/Image:Benkirk_orbiter_schem_upper_tail.png)

Benkirk_orbiter_schem_upper_tail.png

NUMERICAL MATHEMATICS IN DAILY LIFE

- ★ signal (image, audio, and video) processing

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- Golden Delicious ~Mike Doughty
- Shallow Believer [... ~The Used
- I Saw God Today ~George Strait

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1. Low (Feat. T-Pain) by Flo Rida
2. Late 4 Us by Chrisian

Today's Top MP3 Artists

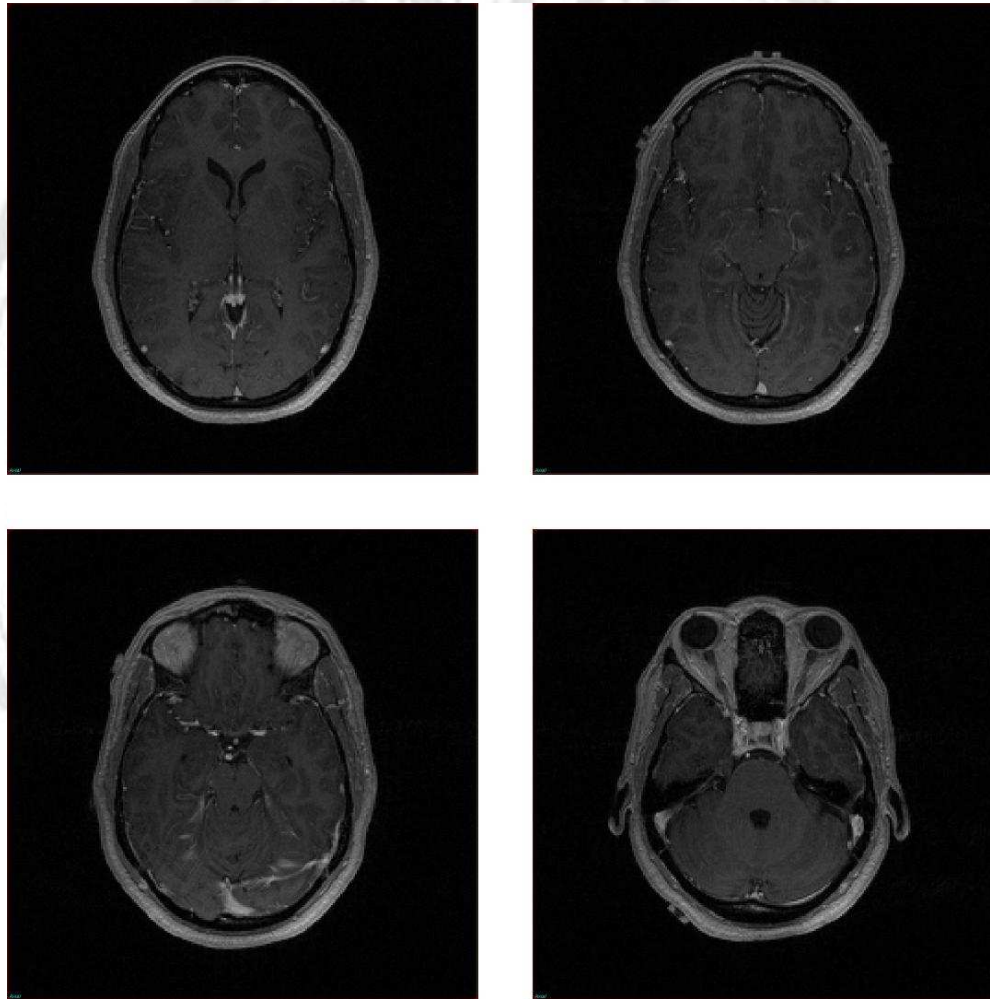
1. Jack Johnson
2. Amy Winehouse

Done

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NUMERICAL MATHEMATICS IN DAILY LIFE

- ★ computer graphics



NUMERICAL MATHEMATICS IN DAILY LIFE

- ★ weather prediction



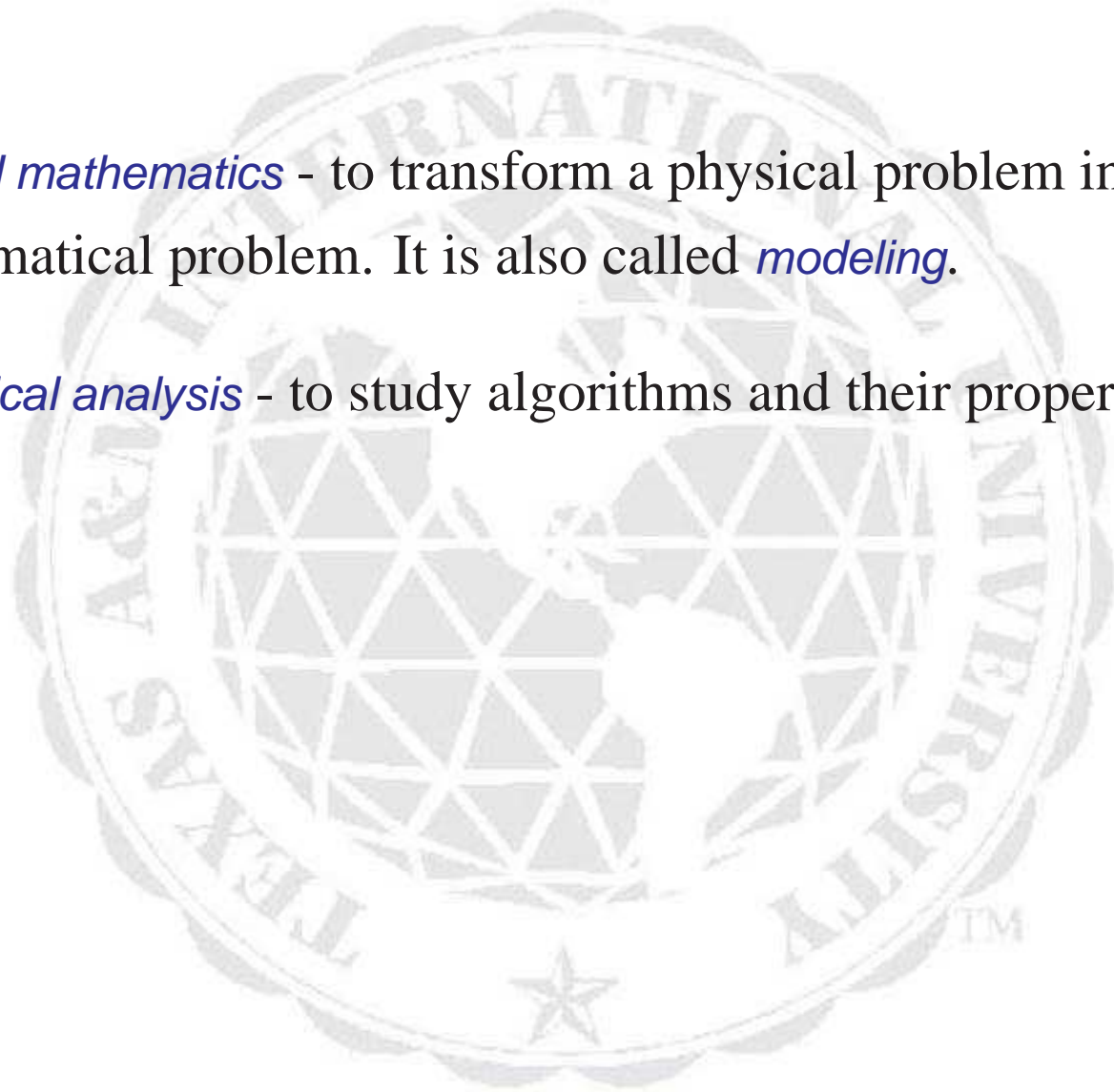
NUMERICAL MATHEMATICS IN DAILY LIFE

- ★ cryptography (e.g. ATM cards, computer passwords, and electronic commerce)



3. SCIENTIFIC COMPUTING IN GENERAL

- ★ *Applied mathematics* - to transform a physical problem into a mathematical problem. It is also called *modeling*.
- ★ *Numerical analysis* - to study algorithms and their properties.



4. EXAMPLES OF CONDUCTED/CONDUCTING PROJECTS

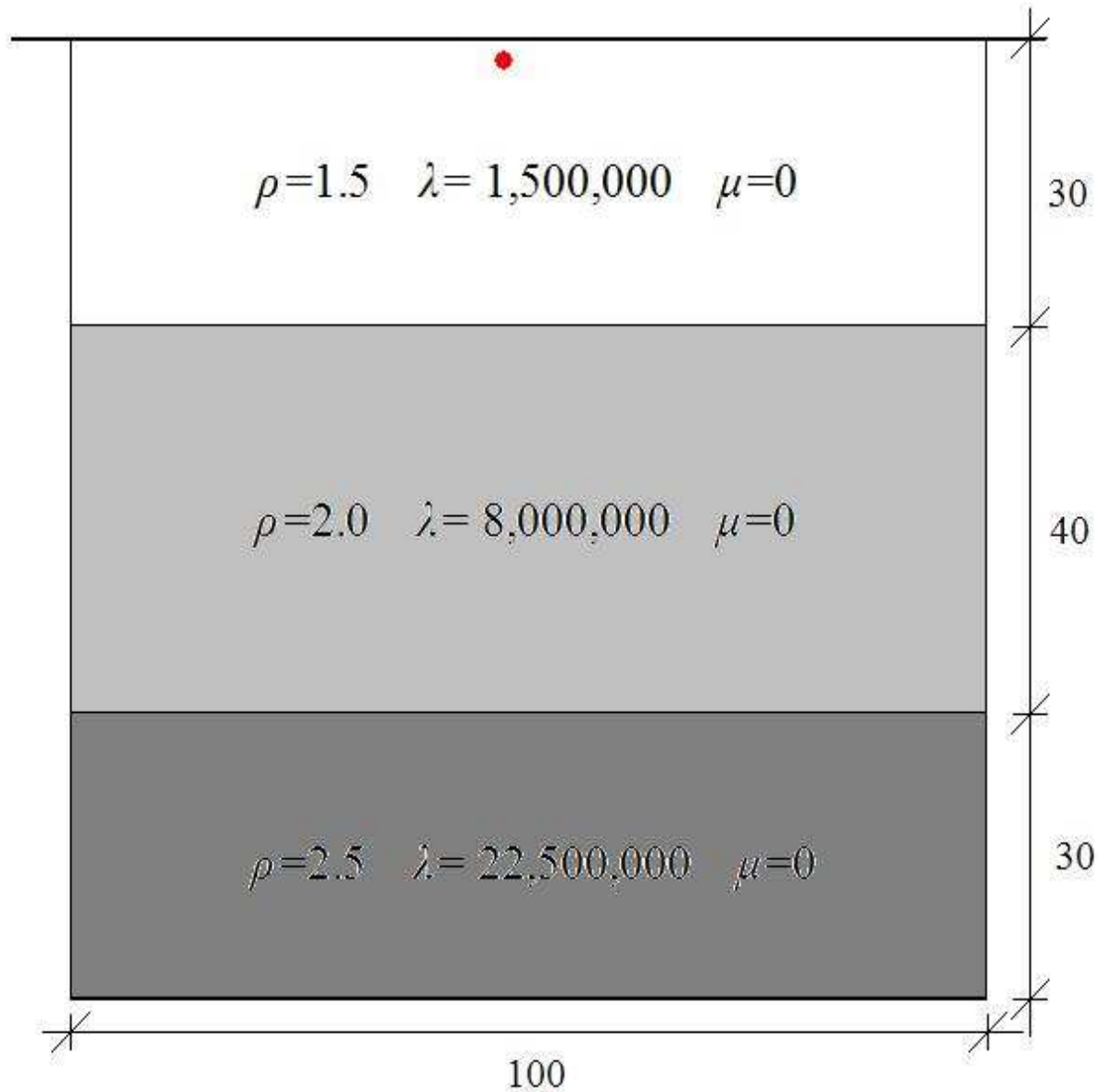
Example 1: Computer simulation of seismic wave propagation

This is a multi-physical process. Here, we consider the elastic wave equations

$$\rho \begin{pmatrix} u \\ v \end{pmatrix}_{tt} = \begin{pmatrix} \lambda + 2\mu & 0 \\ 0 & \mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{xx} + \begin{pmatrix} \mu & 0 \\ 0 & \lambda + 2\mu \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{yy} + \begin{pmatrix} 0 & \lambda + \mu \\ \lambda + \mu & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}_{xy}$$

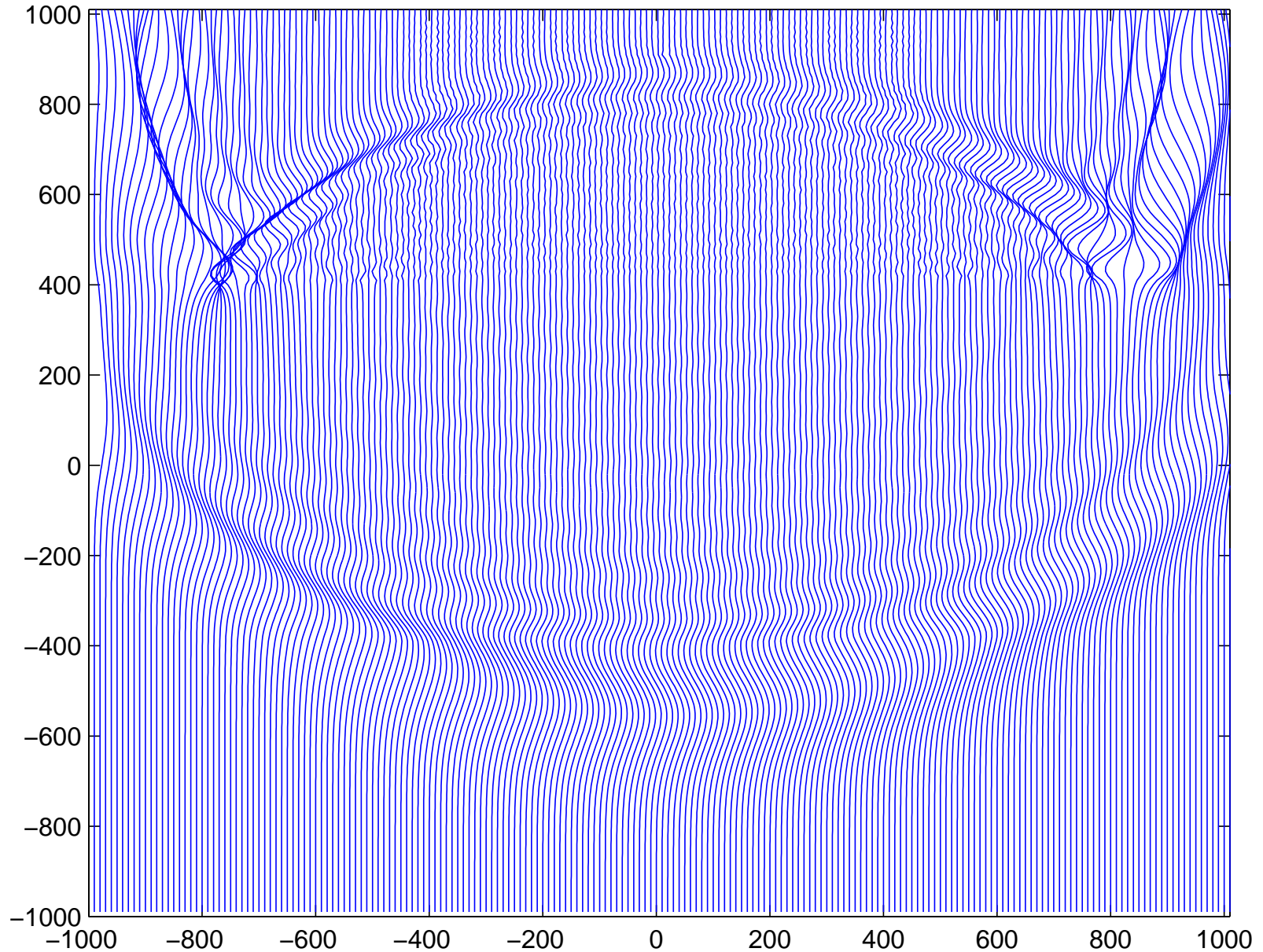
with Clayton-Engquist absorbing boundary conditions. Here $\rho(x, y)$ is the density of the medium, $\lambda(x, y)$ and $\mu(x, y)$ are the Lamé parameters, u and v are the x - and y -displacements, respectively.

Computation region and parameters



snapshot at $t = 1.00\text{s}$

snapshot at $t=1\text{s}$



Example 2: Consider the 1D **singularly perturbed** problem

$$\begin{cases} -\epsilon u''(x) + cu(x) = f(x) & \text{on } (0, 1) \\ u(0) = u(1) = 0 \end{cases} \quad (1)$$

where $0 < \epsilon \ll 1$ is the **perturbation parameter**, $c > 0$ is a constant, the functions f is smooth. The differential equation (1) depends on the ϵ and whose solution approaches a discontinuous limit as $\epsilon \rightarrow 0$.

Exact Solution to a Singularly Perturbed Problem

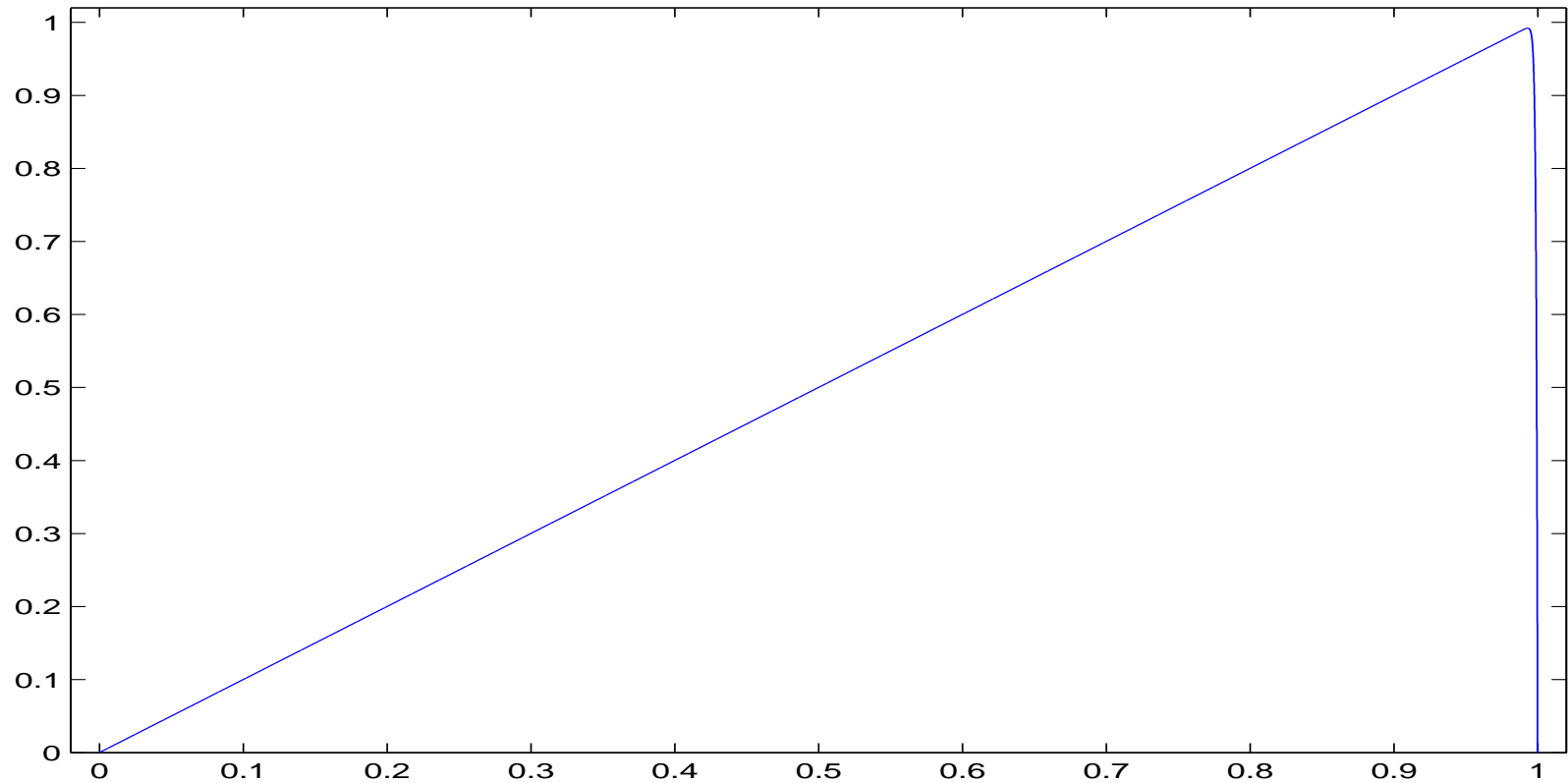
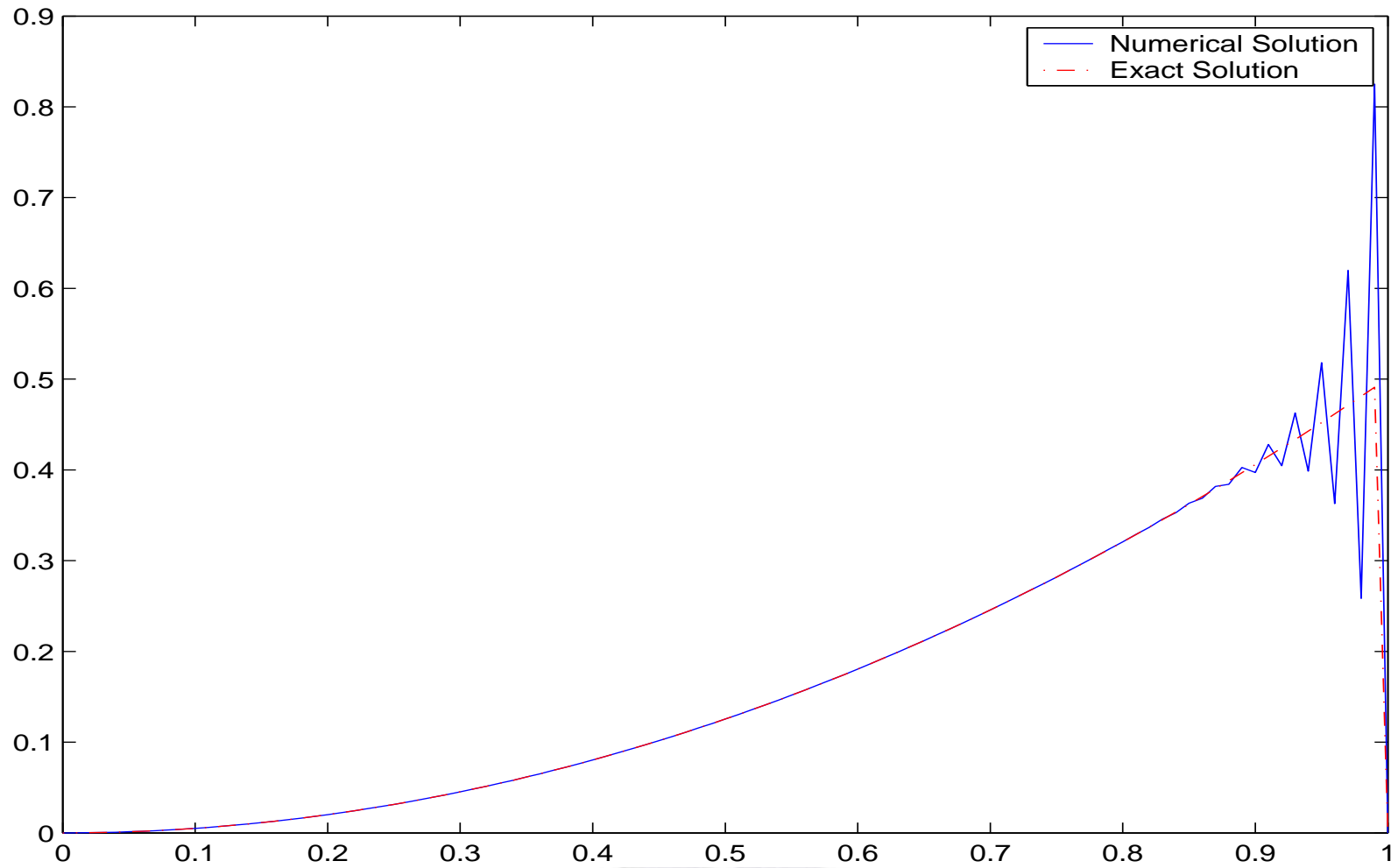
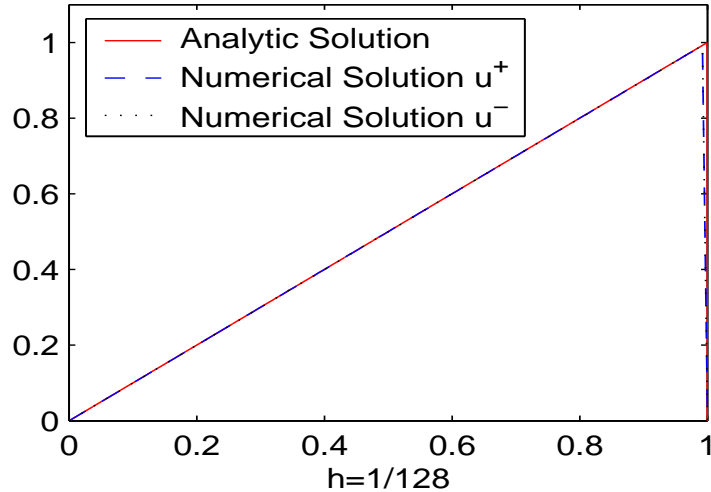
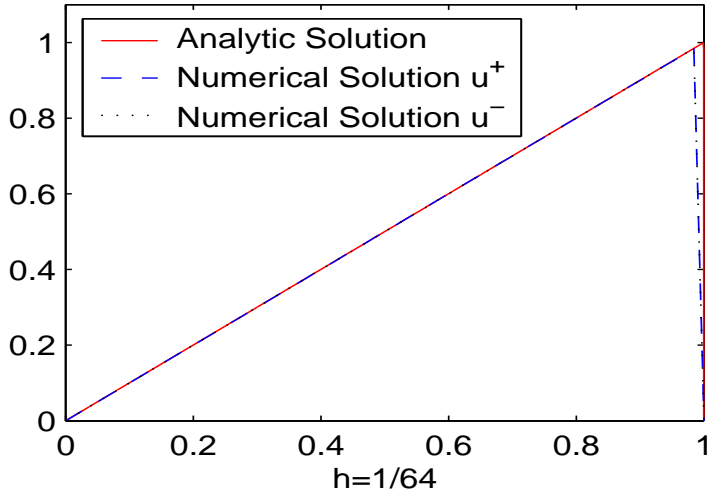
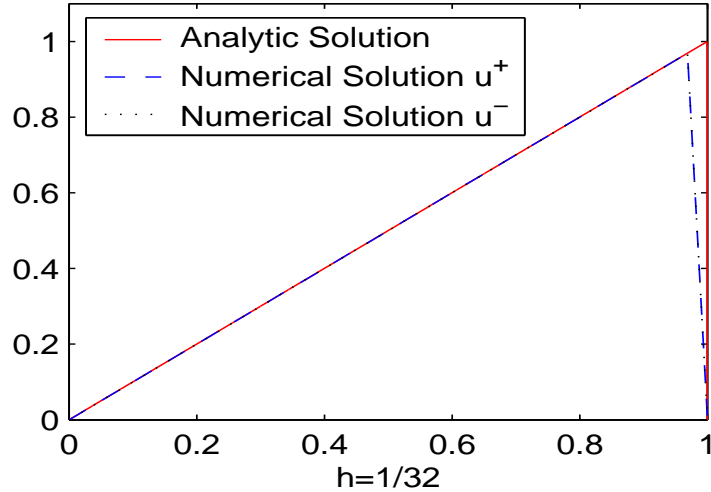
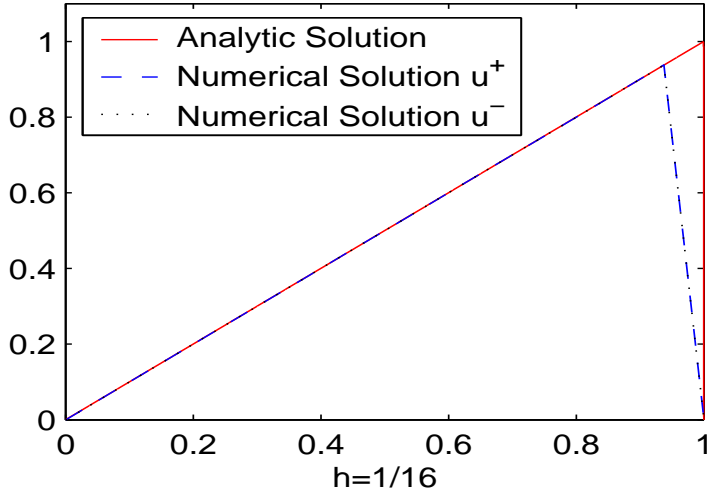


Figure 1. $\epsilon = 10^{-6}$, $c = 1$, $f(x) = x$

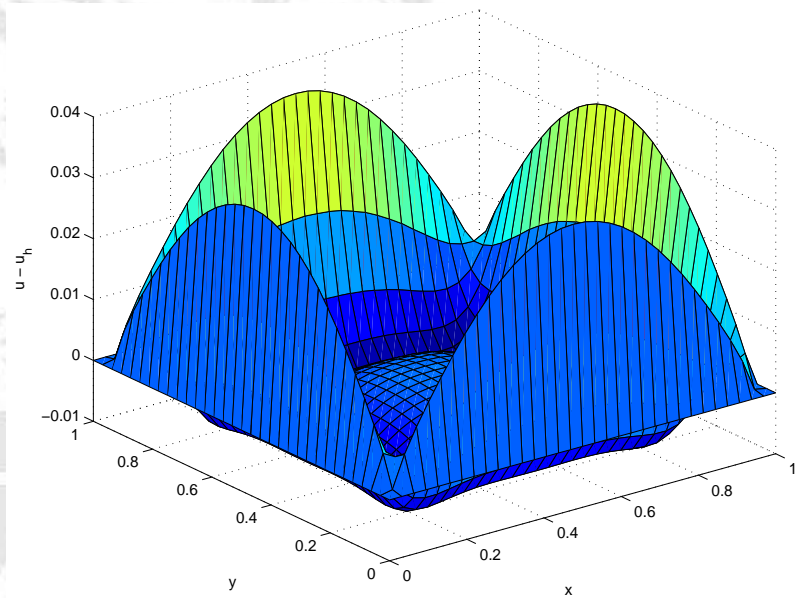
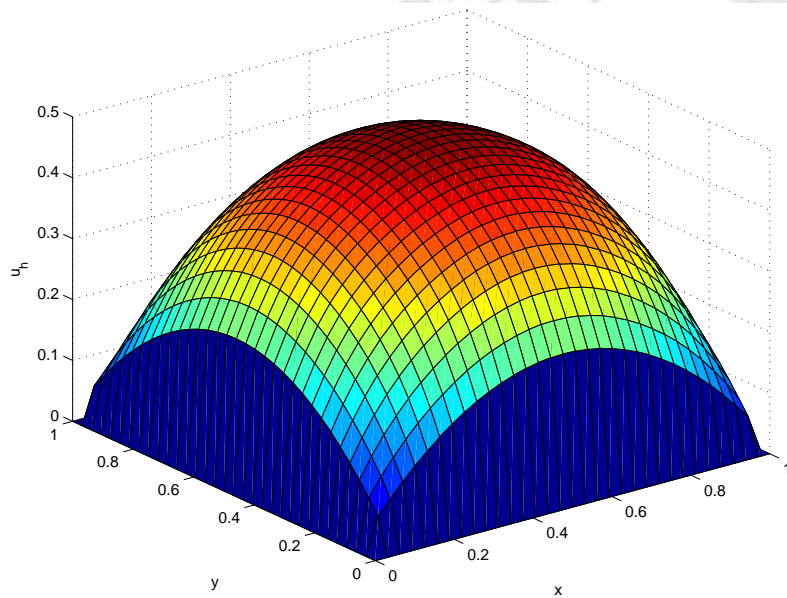
Classical Finite Element Methods Fail



Numerical results by a discontinuous Galerkin least-squares finite element method



A 2D example - numerical solutions and errors



Another 2D example - numerical solutions and errors

