A few limits via cancellations

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The most common limits are computed using a cancellation of like terms in a fraction. The trick is use some kind of algebraic manipulation to get the fraction into a form where cancellation is possible. Here's an easy example.

$$\lim_{x\to-1}\frac{x^2-4x-5}{x+1}=\lim_{x\to-1}\frac{(x+1)(x-5)}{x+1}=\lim_{x\to-1}x-5=-1-5=-6$$

Here's a slightly harder example. You have to combine the top two fractions into a single fraction before cancellation is possible.

$$\lim_{x\to3}\frac{\frac{1}{x-1}-\frac{1}{2}}{3-x}=\lim_{x\to3}\frac{\frac{2}{2(x-1)}-\frac{x-1}{2(x-1)}}{3-x}=\lim_{x\to3}\frac{\frac{2-(x-1)}{2(x-1)}}{3-x}=\lim_{x\to3}\frac{2-(x-1)}{2(x-1)(3-x)}=\lim_{x\to3}\frac{3-x}{2(x-1)(3-x)}$$

$$=\lim_{x\to3}\frac{1}{2(x-1)}=\frac{1}{2(3-1)}=\frac{1}{4}$$

Square roots makes things harder, as we'll see below, but there is a way to deal with them. The trick is to use the identity $(a+b)(a-b)=a^2-b^2$ to remove a square root.

$$\lim_{x\to-2}\frac{\frac{x}{\sqrt{x+11}}+\frac{2}{3}}{x+2}=\lim_{x\to-2}\frac{\frac{3x}{3\sqrt{x+11}}+\frac{2\sqrt{x+11}}{3\sqrt{x+11}}}{x+2}=\lim_{x\to-2}\frac{\frac{3x+2\sqrt{x+11}}{3\sqrt{x+11}}}{x+2}=\lim_{x\to-2}\frac{3x+2\sqrt{x+11}}{3\sqrt{x+11}(x+2)}$$

$$=\lim_{x\to-2}\frac{(3x+2\sqrt{x+11})(3x-2\sqrt{x+11})}{3\sqrt{x+11}(x+2)(3x-2\sqrt{x+11})}=\lim_{x\to-2}\frac{(3x)^2-(2\sqrt{x+11})^2}{3\sqrt{x+11}(x+2)(3x-2\sqrt{x+11})}$$

$$=\lim_{x\to-2}\frac{9x^2-4(x+11)}{3\sqrt{x+11}(x+2)(3x-2\sqrt{x+11})}=\lim_{x\to-2}\frac{9x^2-4x-44}{3\sqrt{x+11}(x+2)(3x-2\sqrt{x+11})}$$

$$=\lim_{x\to-2}\frac{(x+2)(9x-22)}{3\sqrt{x+11}(x+2)(3x-2\sqrt{x+11})}=\lim_{x\to-2}\frac{9x-22}{3\sqrt{x+11}(3x-2\sqrt{x+11})}$$

$$=\frac{9(-2)-22}{3\sqrt{(-2)+11}(3(-2)-2\sqrt{(-2)+11})}=\frac{-40}{3\sqrt{9}(-6-2\sqrt{9})}=\frac{-40}{3(3)-(-6-2(3))}=\frac{-40}{9+6+6}=-\frac{40}{21}$$