221 Calculus, Fall 2007, Section 306/308

Homework 4 (Due in class October 30)

1 Exercise What are the absolute minimum and absolute maximum of $f(x) = x - 2\sin x$ on $[-2\pi, 2\pi]$? At which x in $(-2\pi, 2\pi)$ does $f(x)$ have a local maximum? List two intervals on which $f(x)$ is concave up.

2 Exercise Solve #22 on page 287 of Thomas' *Calculus*.

3 Exercise Calculate the following limit.

$$\lim_{x\to\pi/2} \frac{1}{2x-\pi}\left(\tan x + \frac{2}{2x-\pi}\right)$$

4 Exercise Let $f(x) = x^3 - x + 1$. Prove that $f(x)$ has a root. Then prove that $f(x)$ does not have two roots. Apply three iterations of Newton's method to $f(x)$, starting with $x_0 = -1$. List approximate values (errors not to exceed 10^{-4}) of x_0, $f(x_0)$, x_1, $f(x_1)$, x_2, $f(x_2)$, x_3, and $f(x_3)$. (A calculator or computer is recommended!)

5 Exercise Given $f'(x) = x^3(4 + \sqrt{x}) + 5\sin(x/3)$ and $f(0) = -2$, find $f(x)$.

6 Exercise (Optional) Use Cauchy's Mean Value Theorem to prove that the error in approximating $\sin x$ by x is not greater than $x^3/6$.